

XXIV. *On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of Life Contingencies. In a Letter to FRANCIS BAILY, Esq. F. R. S. &c. By BENJAMIN GOMPERTZ, Esq. F. R. S.*

Read June 16, 1825.

DEAR SIR,

THE frequent opportunities I have had of receiving pleasure from your writings and conversation, have induced me to prefer offering to the Royal Society through your medium, this Paper on Life Contingencies, which forms part of a continuation of my original paper on the same subject, published among the valuable papers of the Society, as by passing through your hands it may receive the advantage of your judgment.

I am, Dear Sir, yours with esteem,

BENJAMIN GOMPERTZ.

9th June 1825.

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## CHAPTER I.

ARTICLE 1. **I**N continuation of Art. 2. of my paper on the valuation of life contingencies, published in the Philosophical Transactions of this learned Society, in which I observed the near agreement with a geometrical series for a short period of time, which must pervade the series which expresses the number of living at ages in arithmetical progression, pro-

ceeding by small intervals of time, whatever the law of mortality may be, provided the intervals be not greater than certain limits: I now call the reader's attention to a law observable in the tables of mortality, for equal intervals of long periods; and adopting the notation of my former paper, considering  $L_x$  to express the number of living at the age  $x$ , and using  $\lambda$  for the characteristic of the common logarithm; that is, denoting by  $\lambda(L_x)$  the common logarithm of the number of persons living at the age of  $x$ , whatever  $x$  may be, I observe that if  $\lambda(L_n) - \lambda(L_{n+m})$ ,  $\lambda(L_{n+m}) - \lambda(L_{n+2m})$ ,  $\lambda(L_{n+2m}) - \lambda(L_{n+3m})$ , &c. be all the same; that is to say, if the differences of the logarithms of the living at the ages  $n, n+m; n+m, n+2m; n+2m, n+3m; \&c.$  be constant, then will the numbers of living corresponding to those ages form a geometrical progression; this being the fundamental principle of logarithms.

Art. 2. This law of geometrical progression pervades, in an approximate degree, large portions of different tables of mortality; during which portions the number of persons living at a series of ages in arithmetical progression, will be nearly in geometrical progression; thus, if we refer to the mortality of DEPARCIEUX, in Mr. BAILY's life annuities, we shall have the logarithm of the living at the ages 15, 25, 35, 45, and 55 respectively, 2,9285; 2,88874; 2,84136; 2,79379; 2,72099, for  $\lambda(L_{15})$ ;  $\lambda(L_{25})$ ;  $\lambda(L_{35})$ ; &c. and we find  $\lambda(L_{25}) - \lambda(L_{15}) = ,04738$   $\lambda(L_{35}) - \lambda(L_{25}) = ,04757$ , and consequently these being nearly equal (and considering that for small portions of time the geometrical progression takes place very nearly) we observe that in those tables the numbers of

living in each yearly increase of age are from 25 to 45 nearly, in geometrical progression. If we refer to Mr. MILNE'S table of Carlisle, we shall find that according to that table of mortality, the number of living at each successive year, from 92 up to 99, forms very nearly a geometrical progression, whose common ratio is  $\frac{3}{4}$ ; thus setting out with 75 for the number of living at 92, and diminishing continually by  $\frac{1}{4}$ , we have to the nearest integer 75, 56, 42, 32, 24, 18, 13, 10, for the living at the respective ages 92, 93, 94, 95, 96, 97, 98, 99, which in no part differs from the table by  $\frac{1}{37}$ th part of the living at 92.

Art. 3. The near approximation in old age, according to some tables of mortality, leads to an observation, that if the law of mortality were accurately such that after a certain age the number of living corresponding to ages increasing in arithmetical progression, decreased in geometrical progression, it would follow that life annuities, for all ages beyond that period, were of equal value; for if the ratio of the number of persons living from one year to the other be constantly the same, the chance of a person at any proposed age living to a given number of years would be the same, whatever that age might be; and therefore the present worth of all the payments would be independent of the age, if the annuity were for the whole life; but according to the mode of calculating tables from a limited number of persons at the commencement of the term, and only retaining integer numbers, a limit is necessarily placed to the tabular, or indicative possibility of life; and the consequence may be, that the value of life annuities for old age, especially where they are

deferred, should be deemed incorrect, though indeed for immediate annuities, where the probability of death is very great, the limit of the table would not be of so much consequence, for the present value of the first payment would be nearly the value of the annuity.

Such a law of mortality would indeed make it appear that there was no positive limit to a person's age; but it would be easy, even in the case of the hypothesis, to show that a very limited age might be assumed to which it would be extremely improbable that any one should have been known to attain.

For if the mortality were, from the age of 92, such that  $\frac{1}{4}$  of the persons living at the commencement of each year were to die during that year, which I have observed is nearly the mortality given in the Carlisle tables between the ages 92 and 99,\* it would be above one million to one that out of three millions of persons, whom history might name to have reached the age of 92, not one would have attained to the age of 192, notwithstanding the value of life annuities of all ages above 92 would be of the same value. And though the limit to the possible duration of life is a subject not likely ever to be determined, even should it exist, still it appears interesting to dwell on a consequence which would follow, should the mortality of old age be as above described. For, it would follow that the non-appearance on the page of history of a single circumstance of a person having arrived

\* If from the Northampton tables we take the numbers of living at the age of 88 to be 83, and diminish continually by  $\frac{1}{4}$  for the living, at each successive age, we should have at the ages 88, 89, 90, 91, 92, the number of living 83; 61.3; 45.9; 34.4; 25.8; almost the same as in the Northampton table.

at a certain limited age, would not be the least proof of a limit of the age of man ; and further, that neither profane history nor modern experience could contradict the possibility of the great age of the patriarchs of the scripture. And that if any argument can be adduced to prove the necessary termination of life, it does not appear likely that the materials for such can in strict logic be gathered from the relation of history, not even should we be enabled to prove (which is extremely likely to be the state of nature) that beyond a certain period the life of man is continually becoming worse.

Art. 4. It is possible that death may be the consequence of two generally co-existing causes ; the one, chance, without previous disposition to death or deterioration ; the other, a deterioration, or an increased inability to withstand destruction. If, for instance, there be a number of diseases to which the young and old were equally liable, and likewise which should be equally destructive whether the patient be young or old, it is evident that the deaths among the young and old by such diseases would be exactly in proportion of the number of young to the old ; provided those numbers were sufficiently great for chance to have its play ; and the intensity of mortality might then be said to be constant ; and were there no other diseases but such as those, life of all ages would be of equal value, and the number of living and dying from a certain number living at a given earlier age, would decrease in geometrical progression, as the age increased by equal intervals of time ; but if mankind be continually gaining seeds of indisposition, or in other words, an increased liability to death (which appears not to be an unlikely supposition with respect to a great part of life, though

the contrary appears to take place at certain periods) it would follow that the number of living out of a given number of persons at a given age, at equal successive increments of age, would decrease in a greater ratio than the geometrical progression, and then the chances against the knowledge of any one having arrived to certain defined terms of old age might increase in a much faster progression, notwithstanding there might still be no limit to the age of man.

Art. 5. If the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, then at the age  $x$  his power to avoid death, or the intensity of his mortality might be denoted by  $aq^x$ ,  $a$  and  $q$  being constant quantities; and if  $L_x$  be the number of living at the age  $x$ , we shall have  $aL_x \times \dot{q} \cdot x$  for the fluxion of the number of deaths  $= -(L_x) \cdot$ ;  $\therefore abq^x = -\frac{\dot{L}_x}{L_x}$ ,  $\therefore abq^x = -\text{hyp. log. of } b \times \text{hyp. log. of } L_x$ , and putting the common logarithm of  $\frac{1}{b} \times$  square of the hyperbolic logarithm of  $10 = c$ , we have  $c \cdot q^x = \text{common logarithm of } \frac{L_x}{d}$ ;  $d$  being a constant quantity, and therefore  $L_x$  or the number of persons living at the age of  $x = d \cdot \overline{g}^q{}^x$ ;  $g$  being put for the number whose common logarithm is  $c$ . The reader should be aware that I mean  $\overline{g}^q{}^x$  to represent  $g$  raised to the power  $q^x$  and not  $g^q$  raised to the  $x$  power; which latter I should have expressed by  $\overline{g}^q{}^x$ , and which would evidently be equal to  $g^{q^x}$ . I take this opportunity to make this observation, as algebraists are sometimes not sufficiently precise in their notation of exponentials.

This equation between the number of the living, and the age, becomes deserving of attention, not in consequence of its hypothetical deduction, which in fact is congruous with many natural effects, as for instance, the exhaustions of the receiver of an air pump by strokes repeated at equal intervals of time, but it is deserving of attention, because it appears corroborated during a long portion of life by experience; as I derive the same equation from various published tables of mortality during a long period of man's life, which experience therefore proves that the hypothesis approximates to the law of mortality during the same portion of life; and in fact the hypothesis itself was derived from an analysis of the experience here alluded to.

Art. 6. But previously to the interpolating the law of mortality from tables of experience, I will premise that if, according to our notation, the number of living at the age  $x$  be denoted by  $L_x$ , and  $\lambda$  be the characteristic of a logarithm, or such that  $\lambda(L_x)$  may denote the logarithm of that number, that if  $\lambda(L_a) - \lambda(L_{a+r}) = m$ ,  $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = mp$ ,  $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = m^2 p$ ; and generally  $\lambda(L_{a+n-r}) - \lambda(L_{a+n}) = m \cdot p^{\frac{n}{r}-1}$ ; that by continual addition we shall have  $\lambda(L_a) - \lambda(L_{a+n}) = m(1 + p + p^2 + p^3 + \dots + p^{\frac{n}{r}-1}) = m \cdot \frac{1-p^{\frac{n}{r}}}{1-p}$ ; and therefore if  $p^{\frac{1}{r}} = q$ , and  $\epsilon$  be put equal to the number whose common logarithm is  $\frac{m}{1-q^n}$ , we shall have  $\lambda(L_{a+n}) = \lambda(L_a) - \lambda(\epsilon) \times (1 - q^n) = \lambda\left(\frac{L_a}{\epsilon}\right) + \lambda(\epsilon) \cdot q^n$ ;  $\therefore L_{a+n} = \frac{L_a}{\epsilon} \times \epsilon^{\lambda^{-1}(q^n)}$ ; and this equation, if for  $a + n$  we write  $x$ , will give  $L_x = \frac{L_a}{\epsilon} \cdot \epsilon^{\lambda^{-1}(q^{-a})} \times q^x$ ; and consequently if  $\frac{L_a}{\epsilon}$  be put

$= d$ , and  $\bar{\varepsilon}^{\bar{q}} = g$ , the equation will stand  $L_x = d \cdot \bar{g}^{\bar{q}^x}$ , and  $\lambda(g) = \lambda(\varepsilon) \times \bar{q}^{-a} = \frac{m \bar{q}^{-a}}{1 - \bar{q}^r}$ ; and I observe that when  $q$  is affirmative, and  $\lambda(\varepsilon)$  negative, that  $\lambda(g)$  is negative. The equation  $L_x = d \cdot \bar{g}^{\bar{q}^x}$  may be written in general  $\lambda(L_x) = \lambda(d) \pm$  the positive number whose common logarithm is  $\{\lambda^2(g) \pm x \lambda(g)\}$ , the upper or under sign to be taken according as the logarithm of  $g$  is positive or negative,  $\lambda^2$  standing for the characteristic of a second logarithm; that is, the logarithm of a logarithm,  $\lambda(q) = \frac{1}{r} \times \lambda(p)$ ,  $\lambda^2(g) = \lambda^2(\varepsilon) - a \cdot \lambda(q) = \lambda\left(\frac{m}{1-p}\right) - a \cdot \lambda(q) = \lambda(m) - \lambda(1-p) - a \lambda(q)$ ; also  $\lambda(d) = \lambda(L_a) - \frac{m}{1-p}$ .

Art. 7. Applying this to the interpolation of the Northampton table, I observe that taking  $a = 15$  and  $r = 10$  from that table, I find  $\lambda(L_a) - \lambda(L_{a+r}) = ,0566 = m$ ,  $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = ,0745$ ,  $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = ,0915$ , and  $\lambda(L_{a+3r}) - \lambda(L_{a+4r}) = ,1228$ ; now if these numbers were in geometrical progression, whose ratio is  $p$ , we should have respectively  $m = ,0566$ ;  $m p = ,0745$ ;  $m p^2 = ,0915$ ;  $m p^3 = ,1228$ . No value of  $p$  can be assumed which will make these equations accurately true; but the numbers are such that  $p$  may be assumed, so that the equation shall be nearly true; for resuming the first and last equations we have  $p^3 = \frac{1228}{566}$ ;  $\therefore$  logarithm of  $p = \frac{1}{3}(\text{logarithm of } 1228 - \text{logarithm of } 566) = ,11213$ ,  $\therefore \lambda(q) = ,011213$  and  $p = 1,2944$ . And to examine how near this is to the thing required, continually to the logarithm of  $,0566$  namely  $\bar{2},75282$ , adding  $,11213$  which is the logarithm of  $p$ , we have respectively for the



logarithms of  $m p$ , of  $m p^2$ , of  $m p^3$  the values  $\bar{2},8649$ ,  $\bar{2},9771$ ,  $\bar{1},0892$ ; the numbers corresponding to which are ,07327; ,09486; ,1228; and consequently  $m$ ,  $m p$ ,  $m p^2$ , and  $m p^3$  respectively equal to ,0566; ,07327; ,09486, and ,1228 which do not differ much from the proposed series ,0566; ,07327; ,09486, and ,1228; and according to our form for interpolation, taking  $m = ,0566$  and  $p = 1,2944$ ; we have  $\frac{m}{1-p} = -\frac{,0566}{,2944} = -,1922$ ; and  $\lambda(L_{15})$  agreeably to the Northampton tables, being = 3,7342 we have  $\lambda(d) = 3,7342 + ,1922 = 3,9264$ ,  $d = 8441$ ,  $\lambda^2(q)$ , that is to say, the logarithm of the logarithm of  $q = \lambda\left(\frac{m}{1-p}\right) - a \lambda(q) = \bar{1},28375 - ,16819 = \bar{1},1156$ ,  $\lambda(g) = -,130949 = \bar{1},8695$ , the negative sign being taken because  $\lambda(g) = \lambda(\epsilon) \times q^{-a} = \frac{m}{1-q} \cdot q^{-a}$ , and  $g = ,7404$ . And therefore  $x$  being taken between the limits, we are to examine the degree of proximity of the equation  $L_x = 8441 \times \sqrt[1,0261^x]{7404}$  or  $\lambda(L_x)$ , that is, the logarithm of the number of living at the age  $x = 3,9264$  — number whose logarithm is ( $\bar{1},11556 + x \times ,011213$ ), as the logarithm of  $g$  is negative. The table constructed according to this formula, which I shall lay before the reader, will enable him to judge of the proximity it has to the Northampton table; but previously thereto shall show that the same formula, with different constants, will serve for the interpolations of other tables.

Art. 8. To this end let it be required to interpolate DEPARCIEUX'S tables, in Mr. BAILY'S life annuities, between the ages 15 and 55.

The logarithms of the living at the age of

15	are 2,92840 differences = ,03966	= $\lambda(L_{15}) - \lambda(L_{25})$
25	2,88874	,04738 = $\lambda(L_{25}) - \lambda(L_{35})$
35	2,84136	,04757 = $\lambda(L_{35}) - \lambda(L_{45})$
45	2,79379	,07280 = $\lambda(L_{45}) - \lambda(L_{55})$
55	2,72099	

Here the three first differences, instead of being nearly in geometrical progression are nearly equal to each other, showing from a remark above, that the living, according to these tables, are nearly in geometrical progression; and the reader might probably infer that this table will not admit of being expressed by a formula similar to that by which the Northampton table has been expressed between the same limits, but putting,

on the supposition of the possibility, though the thing cannot be accurately true,

$$\left\{ \begin{array}{l} \lambda(L_{15}) = \dots \dots \dots = 2,92840 \\ \lambda(L_{25}) = \lambda(L_{15}) - m \dots \dots = 2,88874 \\ \lambda(L_{35}) = \lambda(L_{15}) - m - mp \dots \dots = 2,84136 \\ \lambda(L_{45}) = \lambda(L_{15}) - m - mp - mp^2 \dots = 2,79379 \\ \lambda(L_{55}) = \lambda(L_{15}) - m - mp - mp^2 - mp^3 = 2,72099 \end{array} \right\} \text{ and we shall have}$$

$\lambda(L_{15}) - \lambda(L_{35})$  or its equal  $m + mp = ,08704$ , and  $\lambda(L_{35}) - \lambda(L_{55})$  or its equal  $p^2 \times m + pm = ,12037$ ;  $\therefore p^2 = \frac{12037}{8704}$  and the log. of  $p = \frac{\log. \text{ of } 12037 - \log. \text{ of } 8704}{2} = ,0703997$  and  $p = 1,176$ ,  $m = \frac{,08704}{1 + p} = \frac{,08704}{2,176} = ,04$ . And to see how these values of  $m$  and  $p$  will answer for the approximate determination of the logarithms above set down of the numbers of living at the ages 15, 25, 35, 45, and 55, we have the following easy calculation by continually adding the logarithm of  $p$

Logarithm of $m = \bar{2},6020600$		$\lambda(L_{15}) = 2,92840$
Log. of $p = 0,0703997$	therefore $mp = ,047039$	$-m = - ,04$
Log. of $mp = \bar{2},6724597$	$mp^2 = ,055317$	$\lambda(L_{25}) = 2,88840$
Log. of $mp^2 = 2,7428594$	$mp^3 = ,065051$	$-mp = - ,04704$
Log. of $mp^3 = 2,8128591$		<hr style="width: 100%;"/>
		2,84136
		$-mp^2 = - ,05532$
		<hr style="width: 100%;"/>
		2,78604
		$-mp^3 = - ,06505$
		<hr style="width: 100%;"/>
		2,72099

These logarithms of the approximate number of living at the ages 15, 25, 35, 45 and 55, are extremely near those proposed, and the numbers corresponding to these give the number of living at the ages 15, 25, 35, 45 and 55, respectively, 848; 773,4; 694; 612,3; and 526; differing very little from the table in Mr. BAILY's life annuities; namely, 848; 774; 694; 622 and 526. And we have  $a = 15$ ,  $r = 10$ ,  $m = ,04$ ;  $\lambda(m) = \bar{2},60206$ ;  $1 - p = - ,176$ ;  $\lambda q = \frac{1}{10} \lambda(p) = ,00703997$ ;  $\lambda(g) = \frac{m q^{-a}}{1-p} = - \frac{,04 \times q^{-a}}{,176}$ , and is negative;  $\lambda \lambda(g) = \lambda(,04) - 15 \times ,00704 - \lambda(,176) = \bar{1},25095$ ;  $\lambda(d) = \lambda(L_a) - \frac{m}{1-p} = 2,9284 + ,22727 = 3,1557$ ;  $\therefore \lambda(L_x) = 3,1557 -$  number whose log. is  $(\bar{1},25095 + ,00704 x)$ , for the logarithm of living in DEPARCIEUX' table in Mr. BAILY's annuities, between the limits of age 15 and 55. The table which we shall insert will afford an opportunity of appreciating the proximity of this formula to the table.

Art. 9. To interpolate the Swedish mortality among males between the ages of 10 and 50, from the table in Mr. BAILY's annuities:

Here  $\lambda(L_{10}) = 3,779091$

$\lambda(L_{20}) = 3,746868$  to be assumed  $= \lambda(L_{10}) - m$

$\lambda(L_{30}) = 3,703205$  . . .  $= \lambda(L_{10}) - m - mp$

$\lambda(L_{40}) = 3,648165$  . . .  $= \lambda(L_{10}) - m - mp - mp^2$

$\lambda(L_{50}) = 3,564192$  . . .  $= \lambda(L_{10}) - m - mp - mp^2 - mp^3$

Consequently  $m + mp = \lambda(L_{10}) - \lambda(L_{30}) = ,075886$ , and  $\lambda(L_{30}) - \lambda(L_{50}) = p^2 \times \frac{m + mp}{1 + p} = ,139013$ ; therefore  $p^2 = \frac{,139013}{,075886}$ , and  $\lambda(p) = ,1314468$ ;  $\therefore p = 1,3535$ ;  $m = \frac{,075886}{1 + p} = \frac{,075886}{2,3535}$ ;  $\lambda(m) = \bar{2},5084775$ ;  $m = ,032244$ ;  $a = 10$ ;  $r = 10$ ;  $\lambda(q) = ,01314468$ ;  $\lambda g = \frac{m \cdot q^{-10}}{-,3535}$ , negative;  $\lambda \lambda(g) = \lambda(m) - 10 \lambda(q) - \lambda(,3535) = \bar{2},82861$ ;  $\lambda(d) = \lambda L_a - \frac{m}{1 - p} = 3,779091 + ,091218 = 3,8703$ ; consequently this will give between the ages 10 and 50 of Swedish males,

$\lambda(L_x)$  or the logarithm of the living at the age of  $x = 3,8703 -$  number, whose logarithm is  $(\bar{2},82861 + ,013145 x)$ .

A table will also follow to show the proximity of this with Mr. BAILY's table.

Art. 10. For Mr. MILNE's table of the Carlisle mortality we have, as given by that ingenious gentleman,

- $\lambda(L_{10}) = 3,81023$
- $\lambda(L_{20}) = 3,78462$
- $\lambda(L_{30}) = 3,75143$
- $\lambda(L_{40}) = 3,70544$
- $\lambda(L_{50}) = 3,64316$
- $\lambda(L_{60}) = 3,56146$

And the difference of these will form a series nearly in geometrical progression, whose common ratio is  $\frac{4}{3}$ , and in consequence of this, the first method may be adopted for the

interpolations. Thus because  $\lambda(L_{10}) - (L_{20}) = ,02561$ , the first term of the differences, and  $\lambda(L_{50}) - \lambda(L_{60}) = ,0817$ , the fifth term of the differences: take the common ratio  $= \frac{817}{256} \Big| \frac{1}{2}$ , and  $m = ,0256$ ;  $\therefore \lambda(m) = \bar{2},40824$ . These will give  $\lambda(p) = ,126$ ;  $p = 1,3365$ ;  $a = 10$ ,  $r = 10$ ,  $\lambda(q) = ,0126$ ,  $\lambda(\epsilon) = \frac{m}{1-p} = -\frac{,0256}{,3365}$ ;  $\therefore \lambda(g)$  negative;  $\lambda \lambda g = 2,40824 - \lambda(,3365) - ,126 = \bar{2},75526$ ; and  $\lambda(d) = \lambda(L_{10}) + \frac{,0256}{,3365} = 3,88631$ , and accordingly, to interpolate the Carlisle table of mortality for the ages between 10 and 60, we have for any age  $x$ ,

$$\lambda(L_x) = 3,88631 - \text{number whose logarithm is } (\bar{2},88126 + ,0126 x).$$

Here we have formed a theorem for a larger portion of time than we had previously done. If by the second method the theorem should be required from the data of a larger portion of life, we must take  $r$  accordingly larger; thus if  $a$  be taken 10,  $r = 12$ , then the interpolation would be formed from an extent of life from 10 to 58 years; and referring to Mr. MILNE'S tables, our second method would give  $\lambda(L_x) = 3,89063$  — the number whose logarithm is  $(\bar{2},784336 + ,0120948 x)$ ; this differs a little from the other, which ought to be expected.

If the portion between 60 and 100 years of Mr. MILNE'S Carlisle table be required to be interpolated by our second method, we shall find  $p = 1,86466$ ;  $\lambda(m) = \bar{1},30812$ ;  $m = ,20329$ , &c. and we shall have  $\lambda(L_x) = 3,79657$  — the number whose logarithm is  $(\bar{3},74767 + ,02706 x)$ .

This last theorem will give the numbers corresponding to the living at 60, 80, and 100, the same as in the table; but for the ages 70 and 90, they will differ by about one year:

the result for the age of 70 agreeing nearly with the living corresponding to the age 71; and the result for the age 90, agreeing nearly with the living at the age 89 of the Carlisle tables.

Art. 11. Lemma. If according to a certain table of mortality, out of  $a$ , persons of the age of 10, there will arrive  $b$ ,  $c$ ,  $d$ , &c. to the age 20, 30, 40, &c.; and if according to the tables of mortality, gathered from the experience of a particular society, the decrements of life between the intervals 10 and 20, 20 and 30, 30 and 40, &c. is to the decrements in the aforesaid table between the same ages, proportioned to the number of living at the commencement of those intervals respectively, as 1 to  $n$ , 1 to  $n'$ , 1 to  $n''$ , &c. it is required to construct a table of mortality of that society, or such as will give the above data.

Solution. According to the first table, the decrements of life from 10 to 20, 20 to 30, 30 to 40, &c. respectively, will be found by multiplying the number of living at the commencement of each period by  $\frac{a-b}{a}$ ,  $\frac{b-c}{b}$ ,  $\frac{c-d}{c}$ , &c., and therefore, in the Society proposed, the corresponding decrements will be found by multiplying the number of living at those ages by  $\frac{a-b}{a} n$ ;  $\frac{b-c}{b} n'$ ;  $\frac{c-d}{c} n''$  &c.; and the number of persons who will arrive at the ages 20, 30, 40, &c. will be the numbers respectively living at the ages 10, 20, 30, &c. multiplied respectively by  $\frac{1-n \cdot a + nb}{a}$ ,  $\frac{1-n' \cdot b + n'c}{b}$ ,  $\frac{1-n'' \cdot c + n''d}{c}$ , &c.; hence out of the number  $a$ , living at the age 10, there will arrive at the age 10, 20, 30, 40, 50, &c. the numbers  $\frac{1-n \cdot a + nb}{a}$ ;  $\frac{1-n \cdot a + nb}{a} \times \frac{1-n' \cdot b + n'c}{b}$ ;  $\frac{1-n \cdot a + nb}{a} \times \frac{1-n' \cdot b + n'c}{b} \times \frac{1-n'' \cdot c + n''d}{c}$ ; &c. and the numbers for the intermediate ages must be found by interpolation.

In the ingenious Mr. MORGAN'S sixth edition of PRICE'S Annuities, p. 183, vol. i. it is stated, that in the Equitable Assurance Society, the deaths have differed from the Northampton tables; and that from 10 to 20, 20 to 30, 30 to 40, 40 to 50, 50 to 60, and 60 to 80, it appears that the deaths in the Northampton tables were in proportion to the deaths which would be given by the experience of that society respectively, in the ratios of 2 to 1; 2 to 1; 5 to 3; 7 to 5, and 5 to 4. According to this, the decrements in 10 years of those now living at the ages 10, 20, 30, and 40, will be the number living at those ages multiplied respectively by ,0478; ,0730; ,1024; ,1284; and the deaths in twenty years of those now living at the age of 60, would be the number of those living multiplied by ,3163. And also, taking, according to the Northampton table, the living at the age of 10 years equal to 5675, I form a table for the number of persons living at

the ages . . .	10	20	30	40	50	60	70	80
being . . . .	5675	5403,5	5010	4496	3919	3116	*	1197
and the log. of the number of persons living	3,75612	3,73268	3,69984	3,65283	3,59318	3,49360	*	

Consequently, if  $a = 20$ ,  $r = 10$ , we have  $\lambda(L_{20}) = 3,73268$ ;  $\lambda(L_{40}) = \lambda(L_{20}) - m - mp = 3,65283$ ;  $\lambda(L_{60}) = L_{20} - m - mp - \frac{m p^2}{1+p} - m p^3 = 3,49360$ ;  $m \cdot 1 + p = ,07985$ ; and  $m p^3 \times 1 + p = 3,65283 - 3,49360 = ,15923$ ; hence  $\lambda(p) = \frac{1}{2} \lambda\left(\frac{,15923}{,07985}\right) = ,149875$ ; and  $p = 1,412131$ ;  $\lambda(m) = \lambda(,07985) - \lambda(2,41243) = \bar{2},519874$ ; and  $m = ,033013$ ;  $\therefore \lambda(\epsilon) = \frac{-m}{,412131}$  negative;  $\therefore \lambda(g)$  is negative;  $\lambda \lambda(g) = \lambda m - \lambda,412131 - ,0149875 \times 20 = \bar{2},6051$ ;  $\lambda(d) = \lambda(L_{20}) - \lambda(\epsilon) = 3,73268 - ,080302 = 3,813$  sufficiently near; and our formula for the

mortality between the ages of 20 and 60, which appears to me to be the experience of the Equitable Society, is  $\lambda(L_x) = 3,813$  —the number whose log. is  $(\bar{2}.6051 + ,0149875 x)$ .

This formula will give

At the ages .	10	20	30	40	50	60	70	80
No. of living .	5703,2	5403,5	5007	4496	3862	3116	*	1500
Differs from the } proposed by }	28,2	0	+ 3	0	- 57	0	*	303

In the table of Art. 12, the column marked 1, represents the age; column marked 2, represents the number of persons living at the corresponding age; column marked 3, the error to be added to the number of living deduced from the formula, to give the number of living of the table for which the formula is constructed; column marked 4, gives the error in age, or the quantity to be added to the age in column 1, that would give the number of living in the original table, the same as in column 2. It may be proper to observe, that where the error in column 3 and 4 is stated to be 0, it is not meant to indicate that a perfect coincidence takes place, but that the difference is too small to be worth noticing.



Art. 12.  $\lambda(L) = \lambda(d) - \text{number whose logarithm is } (\lambda^2(g) + x \lambda q)$ .

Northampton.				Deparcieux.			Sweden.			Carlisle.			Formula of supposed experience of the Equitable,						
													Compared with supposed exp.		Compared with Carlisle.				
1	2	3	4	2	3	4	2	3	4	2	3	4	2	3	2	3	4	1	
10							6013	0		6460	0	0	5703	-28	6460				
11							5974	-16	- $\frac{1}{8}$	6427	+4	+ $\frac{1}{8}$	5677		6431				
12							5935	-22	- $\frac{1}{4}$	6393	+7	+ $\frac{1}{4}$	5650		6400				
13							5894	-26	- $\frac{3}{8}$	6358	+10	+ $\frac{3}{8}$	5622		6368 $\frac{1}{2}$	- $\frac{1}{2}$		- $\frac{1}{6}$	
14							5852 $\frac{1}{2}$	-24 $\frac{1}{2}$	- $\frac{1}{2}$	6322	+13	+ $\frac{1}{2}$	5594		6336	-1		- $\frac{1}{3}$	14
15	5423			848	0	0	5810	-22 $\frac{1}{2}$	- $\frac{3}{8}$	6286	+14	+ $\frac{1}{8}$	5564		6302	-2		- $\frac{1}{2}$	15
16	5360	+13	+ $\frac{1}{4}$	841	+1	+ $\frac{1}{4}$	5767	-18	- $\frac{1}{4}$	6248	+13	+ $\frac{1}{4}$	5534		6268 $\frac{1}{2}$	-7 $\frac{1}{2}$		- $\frac{1}{6}$	16
17	5297	+23	+ $\frac{1}{2}$	833 $\frac{1}{2}$	+1 $\frac{1}{2}$	+ $\frac{1}{2}$	5722	-12	- $\frac{1}{2}$	6210	+9	+ $\frac{1}{2}$	5503		6233	-14		- $\frac{1}{3}$	17
18	5233	+29	+ $\frac{3}{4}$	826	+2	+ $\frac{3}{4}$	5677	-6	- $\frac{1}{2}$	6171	+5	+ $\frac{1}{2}$	5470		6196 $\frac{1}{2}$	-20 $\frac{1}{2}$		- $\frac{1}{3}$	18
19	5168	+31	+ $\frac{7}{8}$	819	+2	+ $\frac{7}{8}$	5630	-3	- $\frac{1}{4}$	6131	+2	+ $\frac{7}{8}$	5437		6159	-26		- $\frac{1}{3}$	19
20	5102	+30	+ $\frac{1}{2}$	811	+3	+ $\frac{1}{2}$	5583	-0	0	6090	0	0	5403	0	6120 $\frac{1}{2}$	-30 $\frac{1}{2}$		- $\frac{1}{3}$	20
21	5036	+24	+ $\frac{1}{4}$	804	+2	+ $\frac{1}{4}$	5534	-1	0	6048	-1	0	5368		6081	-34		- $\frac{1}{3}$	21
22	4969	+16	+ $\frac{1}{5}$	796	+2	+ $\frac{1}{5}$	5484	-1	0	6005	+0	0	5333		6040 $\frac{1}{2}$	-35 $\frac{1}{2}$		- $\frac{1}{3}$	22
23	4899	+11	+ $\frac{1}{7}$	789	+1	+ $\frac{1}{7}$	5434	-1	0	5962	+1	0	5295		5998 $\frac{1}{2}$	-35 $\frac{1}{2}$		- $\frac{1}{3}$	23
24	4830	+5	+ $\frac{1}{13}$	781	+1	+ $\frac{1}{13}$	5382	-4	- $\frac{1}{13}$	5917	+4	+ $\frac{1}{10}$	5258		5955 $\frac{1}{2}$	-34 $\frac{1}{2}$		- $\frac{1}{3}$	24
25	4762	-2	- $\frac{1}{3}$	774	+0	0	5329	-6	- $\frac{1}{3}$	5870 $\frac{1}{2}$	+8 $\frac{1}{2}$	+ $\frac{1}{3}$	5218		5911	-32		- $\frac{1}{3}$	25
26	4689	-4	- $\frac{1}{9}$	766	+0	0	5275	-7	- $\frac{1}{9}$	5825	+11	+ $\frac{1}{4}$	5178		5866	-30		- $\frac{1}{3}$	26
27	4616	-6	- $\frac{1}{13}$	757	+1	+ $\frac{1}{8}$	5220	-7	- $\frac{1}{13}$	5777	+16	+ $\frac{1}{4}$	5137		5819	-26		- $\frac{1}{3}$	27
28	4545	-10	- $\frac{1}{7}$	750	+0	0	5164	-6	- $\frac{1}{7}$	5729	+19	+ $\frac{3}{8}$	5095		5771	-23		- $\frac{1}{3}$	28
29	4472	-12	- $\frac{1}{6}$	742	+0	0	5107	-4	- $\frac{1}{4}$	5679	+19	+ $\frac{1}{3}$	5051		5722	-24		- $\frac{1}{3}$	29
30	4403	-22	- $\frac{1}{5}$	734	+0	0	5049	-0	0	5628 $\frac{1}{2}$	+13 $\frac{1}{2}$	+ $\frac{1}{4}$	5007	-3	5671 $\frac{1}{2}$	-29 $\frac{1}{2}$		- $\frac{1}{3}$	30
31	4325	-15	- $\frac{1}{5}$	726	+0	0	4989 $\frac{1}{2}$	-1 $\frac{1}{2}$	- $\frac{1}{7}$	5578	+7	+ $\frac{1}{8}$	4961		5620	-34		- $\frac{1}{3}$	31
32	4250	-15	- $\frac{1}{5}$	718	+0	0	4929	-1	0	5524	+4	+ $\frac{1}{4}$	4914		5567	-39		- $\frac{1}{3}$	32
33	4174	-14	- $\frac{1}{5}$	710	+0	0	4868	-0	0	5470 $\frac{1}{2}$	+1 $\frac{1}{2}$	+ $\frac{1}{7}$	4866		5512 $\frac{1}{2}$	-42 $\frac{1}{2}$		- $\frac{1}{3}$	33
34	4098	-13	- $\frac{1}{5}$	702	+0	0	4805	+3	+ $\frac{1}{10}$	5416	+1	0	4817		5456 $\frac{1}{2}$	-39 $\frac{1}{2}$		- $\frac{1}{3}$	34
35	4021	-11	- $\frac{1}{7}$	694	+0	0	4741	+7	+ $\frac{1}{9}$	5360	+2	+ $\frac{1}{2}$	4767		5399	-37		- $\frac{1}{3}$	35
36	3944	-11	- $\frac{1}{7}$	686	+0	0	4676	+12	+ $\frac{1}{5}$	5303	+4	+ $\frac{1}{4}$	4715		5341	-34		- $\frac{1}{3}$	36
37	3866	-6	- $\frac{1}{12}$	678	+0	0	4611	+17	+ $\frac{1}{12}$	5245 $\frac{1}{2}$	+5 $\frac{1}{2}$	+ $\frac{1}{10}$	4662		5281	-30		- $\frac{1}{3}$	37
38	3788	-3	- $\frac{1}{27}$	669	+2	+ $\frac{1}{4}$	4544	+24	+ $\frac{1}{3}$	5187	+7	+ $\frac{1}{5}$	4608		5219 $\frac{1}{2}$	-25 $\frac{1}{2}$		- $\frac{1}{3}$	38
39	3709	+1	+ $\frac{1}{7}$	661	+3	+ $\frac{1}{3}$	4476	+28	+ $\frac{1}{3}$	5127	+9	+ $\frac{1}{6}$	4553		5157	-21		- $\frac{1}{3}$	39
40	3630	+5	+ $\frac{1}{15}$	653	+4	+ $\frac{1}{5}$	4407	+41	+ $\frac{2}{3}$	5065	+10	+ $\frac{1}{6}$	4496	0	5093	-18		- $\frac{1}{3}$	40
41	3551	+8	+ $\frac{1}{9}$	645	+5	+ $\frac{1}{3}$	4337	+46	+ $\frac{2}{3}$	5004	+5	+ $\frac{1}{3}$	4438		5027	-18		- $\frac{1}{3}$	41
42	3473	+11	+ $\frac{1}{7}$	636	+7	+1	4266	+45	+ $\frac{1}{2}$	4941	-1	0	4379		5060	-20		- $\frac{1}{3}$	42
43	3392	+12	+ $\frac{1}{6}$	628	+8	+1	4194	+37	+ $\frac{1}{2}$	4877	-1	- $\frac{1}{9}$	4319		4892	-23		- $\frac{1}{3}$	43
44	3312	+14	+ $\frac{1}{5}$	619	+10	+1 $\frac{1}{2}$	4121	+30	+ $\frac{2}{3}$	4812	-14	- $\frac{1}{5}$	4257		4882	-24		- $\frac{1}{3}$	44
45	3235	+13	+ $\frac{1}{6}$	611	+11	+1 $\frac{3}{8}$	4047	+24	+ $\frac{3}{8}$	4746	-19	- $\frac{1}{4}$	4194		4751	-24		- $\frac{1}{3}$	45
46	3152	+18	+ $\frac{1}{4}$	603	+12	+1 $\frac{1}{2}$	3973	+18	+ $\frac{2}{3}$	4678	-21	- $\frac{1}{3}$	4130		4678 $\frac{1}{2}$	-21 $\frac{1}{2}$		- $\frac{1}{3}$	46
47	3072	+20	+ $\frac{1}{3}$	594	+13	+1 $\frac{1}{2}$	3897	+14	+ $\frac{1}{2}$	4610	-22	- $\frac{1}{3}$	4065		4604	-16		- $\frac{1}{3}$	47
48	2991	+23	+ $\frac{1}{3}$	586	+13	+1 $\frac{1}{2}$	3821	+10	+ $\frac{1}{2}$	4541	-20	- $\frac{1}{10}$	3998		4529	-8		- $\frac{1}{3}$	48
49	2911	+25	+ $\frac{1}{3}$	577	+13	+1 $\frac{1}{2}$	3744	+7	+ $\frac{1}{2}$	4471	-13	- $\frac{1}{5}$	3931		4452 $\frac{1}{2}$	+5 $\frac{1}{2}$	+ $\frac{1}{17}$	- $\frac{1}{3}$	49
50	2831	+26	+ $\frac{1}{3}$	569	+12	+1 $\frac{1}{2}$	3660	0	0	4400 $\frac{1}{2}$	-3 $\frac{1}{2}$	- $\frac{1}{7}$	3862	+57	4375	+22	+ $\frac{1}{3}$	- $\frac{1}{3}$	50
51	2752	+24	+ $\frac{1}{3}$	560	+11	+1 $\frac{1}{2}$	3587	-16	- $\frac{1}{6}$	4329	+9	+ $\frac{1}{7}$	3793		4296 $\frac{1}{2}$	+31 $\frac{1}{2}$	+ $\frac{1}{2}$	- $\frac{1}{3}$	51
52	2672	+22	+ $\frac{1}{4}$	552	+8	+1	3508	-32	- $\frac{1}{3}$	4256	+20	+ $\frac{1}{3}$	3721		4215	+61	+1	- $\frac{1}{3}$	52
53	2593	+19	+ $\frac{1}{5}$	543	+6	+ $\frac{1}{2}$	3428	-47	- $\frac{1}{3}$	4182	+29	+ $\frac{1}{7}$	3649		4133	+78	+1	- $\frac{1}{3}$	53
54	2514	+16	+ $\frac{1}{5}$	536	+2	+ $\frac{1}{4}$	3348	-62	- $\frac{1}{3}$	4108	+35	+ $\frac{1}{7}$	3575		4050	+93	+1	- $\frac{1}{3}$	54
55	2436	+12	+ $\frac{1}{7}$	526	+0	0				4033	+40	+ $\frac{6}{11}$	3501		3966	+107	+1	- $\frac{1}{3}$	55
56	2358	+8	+ $\frac{1}{10}$							3957	+43	+ $\frac{1}{2}$	3426		3881	+219		- $\frac{1}{3}$	56
57	2280	+4	+ $\frac{1}{20}$							3880	+44	+ $\frac{1}{2}$	3350		3794 $\frac{1}{2}$			- $\frac{1}{3}$	57
58	2206	-4	- $\frac{1}{20}$							3803	+39	+ $\frac{1}{2}$	3273		3706			- $\frac{1}{3}$	58
59	2123	-3	- $\frac{1}{30}$							3724	+25	+ $\frac{1}{4}$	3195		3169			- $\frac{1}{3}$	59
60	2052	-14	- $\frac{1}{5}$							3646	-3	0	3116	0	3529			- $\frac{1}{3}$	60
							$\lambda(d) = 3,9265$			$\lambda(d) = 3,89063$			$\lambda(d) = 3,813$						
							$\lambda^2(g) = 1,11556$			$\lambda^2(g) = 2,82861$			$\lambda^2(g) = 2,784336$						
							$\lambda(q) = ,011213$			$\lambda(q) = ,0131457$			$\lambda(q) = ,0149875$						
							$\lambda(d) = 3,1557$			$\lambda(d) = 3,8703$			$\lambda(d) = 3,86743$						
							$\lambda^2(g) = 1,25095$			$\lambda^2(g) = 2,82861$			$\lambda^2(g) = 2,6051$						
							$\lambda(q) = ,00704$			$\lambda(q) = ,0131457$			$\lambda(q) = ,0149875$						

## CHAPTER II.

ARTICLE 1. The near proximity to the geometrical progression of the series expressing the number of persons living at equal small successive intervals of time during short periods, out of a given number of persons living at the commencement of those intervals, affords a very convenient mode of calculating values connected with life contingencies, for short limited periods; by offering a manner of forming general tables, applicable (by means of small auxiliary tables of the particular mortalities) to calculations for any particular mortality; and by easy repetition, to calculate the values for any length of period for any table of mortality we please.

If, for instance, it were required to find the value of an annuity of an unit for  $p$  years, on three lives of the age  $b, c, d$ , the rate of interest being such that the present value of an unit to be received at the expiration of one year, be equal to  $r$ , then the

value of the first payment would be  $\frac{L_{b+1}}{L_b} \times \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} \times r$ ;

and of the  $p^{\text{th}}$  payment the present value would be  $\frac{L_{b+p}}{L_b} \times$

$\frac{L_{c+p}}{L_c} \times \frac{L_{d+p}}{L_d} \times r^p$ ; but if  $L_{b+p} = L_b \times \left(\frac{L_{b+1}}{L_b}\right)^p$  whether  $p$  be

1, 2, 3, &c. which will be the case when  $L_b, L_{b+1}, L_{b+2},$  &c. form a geometrical progression, and similarly, if  $L_{c+p} =$

$L_c \times \left(\frac{L_{c+1}}{L_c}\right)^p$ , and also,  $L_{d+p} = L_d \times \left(\frac{L_{d+1}}{L_d}\right)^p$ , the pre-

sent value of the  $p^{\text{th}}$  payment will be  $\left(\frac{L_{1:b,c,d}}{L_{b,c,d}} r\right)^p$ ; hence, if  $\frac{L_{1:b,c,d}}{L_{b,c,d}} r$  be put  $= a$ , the value of the annuity will be  $a + a^2 + a^3 + a^4 \dots a^p = \frac{a-a^{p+1}}{1-a} = \frac{1-a^{p+1}}{a-1}$ .

Art. 2. Consequently, let a general table be formed of the logarithm of  $\frac{1-a^p}{a-1}$  for every value of the log. of  $a^p$ ; and also let a particular table be formed for every value of the log. of  $\frac{L_{x+p}}{L_x}$  according to the particular table of mortality to be adopted; from the last table take the log. of  $\frac{L_{b+p}}{L_b}$ ,  $\frac{L_{c+p}}{L_c}$ ,  $\frac{L_{d+p}}{L_d}$ ; and also from a table constructed for the purpose, take the log. of  $r^p$ , add these four logs. together, and the sum will be the log. of  $\overline{a}^p$ , which being sought for in the general table, will give the log. of  $\left(\frac{1-a^p}{a-1}\right)$  which will be the log. of the annuity sought for the term  $p$ , on supposition of the geometrical progression being sufficiently near. Here I remark, that were it not for more general questions than the above, it would be preferable to have general tables formed for the values of  $\frac{1-a^p}{a-1}$ , instead of the log. of such values; but from the consideration that for most purposes a table of the logs. of  $\frac{1-a^p}{a-1}$  will be found most convenient, I have had them calculated in preference.

Art. 3. The shorter the periods are, the nearer does the series of the number of persons living at the equal intervals of successive ages approximate to the geometrical progression; and consequently this mode, by the assumption of sufficiently short periods, and frequent repetitions, will answer

for any degree of accuracy the given table of mortality will admit of, but then the labour will be increased in proportion.

Art. 4. There are different modes of obviating, in a great measure, this inconvenience, by assuming an accommodated ratio for the given age, instead of the real ratio, from amongst which I shall only for the present select a few. The first is

as follows : find for every value of  $a$ , the log. of  $\frac{1}{p} \int_1^y \frac{L_{x+y}}{L_x}$ , that

is, the log. of  $\frac{L_{x+1}}{L_x} + \frac{L_{x+2}}{L_x} + \frac{L_{x+3}}{L_x} \dots \dots \frac{L_{x+p}}{L_x}$ ; seek

this value in the general table, which will give the corresponding value of the log. of  $a^p$ ; and construct a table of such values for every value of  $x$ , and adopt these values for log. of  $a^p$ , instead of the abovenamed values of the log. of  $\frac{L_{x+p}}{L_x}$ ,

for the determination of the values of the limited periods: the preference of this to the first proposed method consists in this; that if the series  $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} \dots L_{b+p}) =$

$\epsilon + \epsilon^2 + \epsilon^3 + \dots \epsilon^p$ , the series  $\frac{L_{b+1}}{L_b}, \frac{L_{b+2}}{L_b}, \&c.$  being nearly

in geometrical progression, and  $\frac{L_{b+1}}{L_b} - \epsilon = \epsilon_1, \frac{L_{b+2}}{L_b} - \epsilon^2 = \epsilon_2,$   
 $\&c. \epsilon_1, \epsilon_2, \&c.$  will be small, and  $\epsilon_1 + \epsilon_2 + \epsilon_3 \dots \epsilon_p = 0$ , and there-

fore, if the series  $\frac{L_{c+1}}{L_c}, \frac{L_{c+2}}{L_c}, \frac{L_{c+3}}{L_c}, \&c.$  and  $\frac{L_{d+1}}{L_d}, \frac{L_{d+2}}{L_d}, \&c.$

formed accurately geometrical progressions, and the value of  $\frac{L_{c+1} \times L_{d+1}}{L_c \times L_d} . r = m$ , the value of the annuity for the term,

would be accurately equal to  $m \epsilon + m^2 \epsilon^2 + m^3 \epsilon^3 \dots \dots + m^p \epsilon^p + m \epsilon_1 + m^2 \epsilon_2 + m^3 \epsilon_3 \dots \dots + m^p \epsilon_p$ , but because in

general  $\frac{L_{c+1}}{L_c}$ ,  $\frac{L_{d+1}}{L_d}$  and  $r$  differ very little from unity,  $m$  will not differ much from unity; and therefore if  $p$  be not great,  $m$ ,  $m^2$ ,  $m^3$ , &c. will not differ much from unity; and consequently, as  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$ , &c. are small,  $m\epsilon_1 + m^2\epsilon_2 + m^3\epsilon_3 \dots m^p\epsilon_p$  will not differ much from  $\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_p$ ; but this has been shown to be 0; consequently  $m\epsilon_1 + m^2\epsilon_2 + m^3\epsilon_3 \dots + m^p\epsilon_p$  differs very little from 0, or in other words is very small; and consequently, the value of the annuity differs very little from  $m\epsilon + m^2\epsilon^2 + m^3\epsilon^3 \dots + m^p\epsilon^p$ ; and the same method of demonstration would apply with any one of the other ages, the remaining ages being supposed to possess the property of the accurate geometrical progression; notwithstanding this, however, as none of them probably will contain that property, but in an approximate degree, a variation in the above approximations may be produced of a small quantity of the second order; that is, if the order of the product of two small quantities; but, as in this approximation, I was only aiming at retaining the quantities of the first order, I do not consider this as affecting the result as far as the approximation is intended to reach: thus far with regard to the first accommodated ratios.

Art. 5. Moreover, on the supposition that  $L_c$ ,  $L_{c+1}$ ,  $L_{c+2}$ ,  $\dots$ ,  $L_{c+p}$ , and also  $L_d$ ,  $L_{d+1}$ ,  $L_{d+2}$   $\dots$ ,  $L_{d+p}$  are series in geometrical progression, and that  $r \cdot \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} = m = n.q.$  Since the annuity for  $p$  years on the three lives is equal to  $\frac{L_{b+1}}{L_b} \cdot m + \frac{L_{b+2}}{L_b} \cdot m^2 + \dots + \frac{L_{b+p}}{L_b} \cdot m^p$  it follows

that if  $\frac{L_{b+1}}{L_b}.n + \frac{L_{b+2}}{L_b}.n^2 + \frac{L_{b+3}}{L_b}.n^3 \dots \dots \dots \frac{L_{b+p}}{L_b}.n^p =$   
 $\xi.n + \xi^2.n^2 + \xi^3.n^3 \dots \dots + \xi^p.n^p$  that if  $n$  be very nearly equal  
to  $m$ ,  $\frac{L_{b+1}}{L_b}.n.q + \frac{L_{b+2}}{L_b}.n^2.q^2 + \&c. \dots \dots \dots \frac{L_{b+p}}{L_b}.n^p.q^p$  which  
will be the value of the annuity on the three lives, will be  
nearly  $= \xi.n.q + \xi^2.n^2.q^2 + \&c. \dots \dots \xi^p.n^p.q^p$ . If  $q$  were  
equal to unity, or, which is the same thing,  $m=n$ , the  
equality would be accurate; but it may not be so when  $m$   
differs from 1; but the nearer  $n$  is to  $m$ , at least when the  
difference does not exceed certain limited small quantities,  
the nearer will be the coincidence. It appears therefore,  
that if instead of taking the accommodated ratio for  $\xi^p$  so that  
 $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} \dots L_{b+p}) = \xi + \xi^2 + \xi^3 \dots \xi^p$   
it will be preferable generally to take it so that  $\frac{1}{L_b} \times (n.L_{b+1} +$   
 $n^2.L_{b+2} + n.L_{b+3} \&c. \dots n^p.L_{b+p}) = \xi + \xi^2 + \xi^3 \&c. \dots \xi^p$  in  
which  $n$  is between  $m$  and 1, the nearer  $m$  the better generally,  
though possibly not universally so throughout the whole limit.  
And the second method I use for increasing the accuracy, is to  
adopt an accommodated ratio, or  $\xi^p$ , so that  $\frac{1}{L_b} \times (1,05^{-1}L_{b+1} +$   
 $1,05^{-2}L_{b+2} + \&c. \dots 1,05^p L_{b+p}) = 1,05^{-1} \xi + 1,05^{-2} \xi^2 + 1,05^{-3} \xi^3$   
 $\dots 1,05^{-p} \xi^p$ . Another method which might have its peculiar  
advantage, is to assume  $\xi^p = \frac{L_{b+\frac{1}{2}p}}{L_b}$  under the idea of using  
a mean ratio.

The General Tables.\*

Art. 6. I have had three general tables calculated for  
fixed periods, Numbers 1, 2, and 3. Number 1, for pe-

\* The chief of the arithmetical operations in the constructions of most of the  
tables were performed under my direction, by Mr. DAVID JONES, of N<sup>o</sup>. 10, King-  
street, Soho; and, as far as my leisure would allow, I have endeavoured to assure  
myself of their accuracy by different inspections.

riods of ten years; that is, for  $\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$ , corresponding to a given value of  $\lambda (a^{10})$ . N<sup>o</sup>. 2, for seven years, or for  $\lambda \left( \frac{1-a^7}{a^{-1}-1} \right)$ , corresponding to  $\lambda (a^7)$ , and the 3d for five years, or for  $\lambda \left( \frac{1-a^5}{a^{-1}-1} \right)$ , corresponding to  $\lambda (a^5)$ ; calculated (whether  $p = 10, 7$  or  $5$ ) for every value of  $\lambda (a^p)$ , answering to  $\bar{3},00$ ;  $\bar{3},01$ ;  $\bar{3},02$ , &c. . . . 0. The first column containing the aforesaid value of  $\lambda (a^p)$ , corresponding to which, in an horizontal line, is placed the log. of  $\frac{1-a^p}{a^{-1}-p}$ , and between each successive value is placed the difference, retaining a decimal figure more; at the head of the other columns for the proportional parts of the differences, are placed a column showing the number of cyphers to be prefixed to the differences entered in the column following, which are headed

$\left\{ \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 9 & 8 & 7 & 6 & 5 \end{matrix} \right\}$  nearest under 1, 2, 3, 4 and 5, and opposite the number; suppose  $\bar{2},16$  of table log. of  $a^{10}$ , stands

$\left. \begin{matrix} 0275, 0550, 0826, 1101, \\ 2477, 2202, 1926, 1651, \end{matrix} \right\}$  the upper, with the addition of the two cyphers, give the proportional parts for ,001, 002, 003, ,004, 005: and the under, with the two cyphers, shows the proportional parts for ,009, 008, 007, 006; and the reason of choosing this arrangement, is the advantage which it offers of proof of correctness; thus the sum of the higher and lower numbers of each of the above row with the two cyphers = 002752, which is double ,001376, and equal to the whole difference between the successive terms.

Let it be required to find the logarithm of  $\left( \frac{1-a^{10}}{a^{-1}-1} \right)$ , corresponding to log. of  $a^{10} = \bar{1}.7954$ . In the General Table I,

Opposite to $\bar{1}.79$ we have . . .	,88868
For ,005 we have proportional part . . .	256
For ,0004 . . . ditto . . .	20

The sum . ,89144 is the answer.

If log. of  $a^p$  is less than  $\bar{3},00$ , then it will be necessary to calculate  $\lambda\left(\frac{1-a^p}{a^p-1}\right)$  by common methods, as the tables do not go lower. And generally it will be then sufficient, omitting  $a^p$ , only to calculate the value of  $-\lambda(a^{-1}-1)$ ; but from this, if more accuracy be required, subtract the number whose common logarithm is  $(\bar{1},6378 + \lambda(r)^p)$ .

If  $\lambda\left(\frac{1-a^p}{a^p-1}\right)$  be given, and  $\lambda(a)$  be required, proceed thus,  $\lambda\left(\frac{1-a^{10}}{a^{10}-1}\right)$  being = ,89144 for example. In Table I, the next value of

$\lambda\left(\frac{1-a^{10}}{a^{10}-1}\right)$  is ,88868 to which  $\lambda(a^{10})$  corresponding is  $\bar{1},79$

Difference ,00256 belonging to  $\bar{1},79$  gives . . . ,005

Difference ,00020 . . . ditto . . . . ,0004

$\therefore$  if  $\lambda\left(\frac{1-a^{10}}{a^{10}-1}\right) = ,89144$  then we have  $\lambda(a^{10}) = \bar{1},7954$

If  $\lambda(a^p)$  is less than  $\bar{3}$ , proceed thus: put the given value of  $\lambda\left(\frac{1-a^p}{a^p-1}\right) = \lambda q$ , and we have the common logarithm of  $a = -p \times \lambda(1 + q^{-1}) +$  a small correction if great accuracy be required; which correction is nearly equal to

$p \times$  the number whose common log. is  $\{1,6378 - \lambda q^{-p+1} \cdot (1 + q^{-1})\}$

These methods and tables only apply immediately to  $\lambda\left(\frac{1-a^p}{a^p-1}\right)$  when  $a$  is a proper fraction; but if  $a$  be greater than unity, put it equal to  $b^{-1}$ , then will  $b$  be a proper fraction;

but  $\frac{1-a^p}{a^p-1} = \frac{a^p-1}{1-a^p} = \frac{b^{-p}-1}{1-b^{-p}} = \frac{b^{-p-1}-b^p}{b^{-1}-1} = a^{p+1} \times \left(\frac{1-b^p}{b^{-1}-1}\right)$ ; conse-

quently  $\lambda\left(\frac{1-a^p}{a^p-1}\right) = \overline{p+1} \cdot \lambda(p) + \lambda\left(\frac{1-b^p}{b^{-1}-1}\right)$  I have likewise

had Table IV. calculated, which is a general table, for the com-

mon log. of  $\left(\frac{1}{a^{-1}-1}\right)$ , corresponding to a given value of  $\lambda a$ ,



commencing with  $\lambda(a) = 1.7; 1,701; 1,702, \&c.$  with the differences between them. I have not, in this table, had the proportional parts inserted, though it would be attended with advantage, as the table is not meant to be of general use; but only given to be applied for rough purposes, or where accuracy is not particularly required for calculating at once the value of a life annuity for the whole term of life, or the whole remaining terms of life, after a given term, by considering the present value of each successive payment to form the successive terms of a geometrical progression whose first term and common ratio are each equal to  $a$ . And as  $\lambda\left(\frac{1}{a-1}\right)$  will represent the log. of the sum of the said geometrical progression, it will likewise express approximatively the logarithm of the value required. For many purposes, a table of  $\frac{1}{a-1}$ , answering to given values of  $a$ , would be preferable, but not for general purposes.

Art. 7. I have already, in Art. 4 and 5, Chap. II, introduced the term accommodated ratios, or chances, and endeavoured to explain the methods to be adopted to reap the advantage of the ideas there expressed. Table V, for Carlisle, Deparcieux, and Northampton, are the logarithms of tenth terms of the accommodated ratios, or the logarithms of the accommodated chances for living ten years, calculated according to a mode laid down in Art. 5, Chap. II; that is, it expresses for every age, or value of  $b$ , the logarithm of  $\epsilon^{10}$ , when  $\frac{1}{L_b} \times (1,05^{-1} L_{b+1} + 1,05^{-2} L_{b+2} + \&c. \dots 1,05^{-p} L_{b+p})$  is equal to  $1,05^{-1} \epsilon + 1,05^{-2} \epsilon^2 + \&c. \dots 1,05^{-10} \epsilon^{10}$ . and to show, by example, how these are calculated, let it be required to find the logarithm of the accommodated chance for living

ten years, for the age 20, calculated according to the Carlisle table upon the consideration of interest at 5 per cent. Accord-

ing to the Carlisle tables, I find  $\lambda_{10}^{\frac{1,05^{-1}}{1}} \overline{20}$ ; that is, the logarithm of the annuity of one pound on a life of 20, for ten years, at 5 per cent = ,87176, and putting  $a = 1,05^{-1} \cdot \beta$ , by hypothesis

we shall have  $\lambda_{10}^{\frac{x}{1}} a^x$ ; that is the logarithm of  $(a + a^2 + a^3 \dots a^{10}) = ,87176$ ; that is,  $\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right) = ,87176$ ; hence proceeding, as shown above, to find from General Table I.  $\lambda(a^{10})$

Having given . . . ,87176 =  $\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$

We have next less = ,86842 corresponding to . . . . 1.75

,00334 difference

,00302 proportional part . . . . ,006

30 . . ditto . . . . . ,0006

2 . . ditto . . . . . ,00004

,87176 corresponds to .  $\lambda(a^{10}) = \overline{1.75664}$   
 $\lambda(1,05^{10}) = \underline{\underline{,21189}}$

log. of the accommodated chance to live 10 years at the Carlisle mortality.  $\overline{1.96853}$  for the

In the same way may the accommodated chance be found for any other term, when general tables for the term are constructed, and from any other base of interest. I may observe, that by using different rates of interest, as a base for determining the accommodated chances, different degrees of accuracy may be obtained. See Art. 5. Chap. II.

Art. 8. Table VI. is the logarithm of the accommodated chances  $\mathcal{E}$  at every age,  $b$  for living one year, where  $\mathcal{E}$  is of such value that the sum of the geometrical progression  $\frac{\mathcal{E}}{1,05} + \frac{\mathcal{E}^2}{1,05^2} + \&c.$  ad infinitum, or, which is the same thing,

$\frac{1}{\frac{1.05}{6}-1}$  shall be equal to the value of the whole life annuity at five per cent. at such age, namely  $\overset{1.05^{-1}}{\underset{1}{\lceil} b}$ ; consequently  $\frac{6}{1.05^{-1}} \times$

$$\left(1 + \overset{1.05^{-1}}{\underset{1}{\lceil} b}\right) = \overset{1.05^{-1}}{\underset{1}{\lceil} b}; \therefore \lambda \text{ } \mathcal{E} = \lambda \left(\overset{1.05^{-1}}{\underset{1}{\lceil} b}\right) + \lambda(1.05) - \lambda \left(\overset{1.05^{-1}}{\underset{0}{\lceil} b}\right).$$

This table is constructed for Carlisle, Deparcieux, and Northampton, and is to be used in conjunction with Table IV., where only a rough value of the contingency is required; and though this table applies as the other tables of accommodated chances, to different rates of interest, still it would be of advantage more particularly *here* for the greater approximation to have similar tables constructed from the

formula  $\lambda(\mathcal{E}) = \lambda \left(\overset{r}{\underset{1}{\lceil} b}\right) + \lambda(r^{-1}) - \lambda \left(\overset{r}{\underset{0}{\lceil} b}\right)$  for different values of  $r$ .

Art. 9. In calculating the value of life annuities for long periods, by means of adding together the values of portions of those periods, the portions of the distant periods contain factors of the real chance of living to these periods, and likewise of the discounted value of the money of which the payment is not immediate; thus if  $t$  be greater than 10,

$$\overset{r}{\underset{t}{\lceil} a, b, c} = \overset{r}{\underset{10}{\lceil} a, b, c} + \overset{r}{\underset{t}{\lceil} a, b, c} = \overset{r}{\underset{10}{\lceil} a, b, c} + \frac{L_{a, b, c}}{L_{a, b, c}} \cdot r^{10} \times$$

$$\overset{r}{\underset{(t-10)}{\lceil} a+10, b+10, c+10}.$$

It will be therefore convenient to have a table of the logarithm of the real chance of living 10, 20, 30 years, &c. and also for other terms; and some of these are given by Tables VII., VIII., IX.

Time will not allow me, for the present, to offer more than a very few examples of the method to be employed in calculating by these tables, which are as follow :

Example 1. Required, according to the Carlisle table, the value of a life annuity, for ten years, on the joint lives 30 and 40, at 3 per cent interest.

In Table VIII. for Carlisle, log. of accommodated		
chance for 10 years, at the age 30 . . .	=	$\bar{1}.9552$
Ditto 40 . . . . .	=	$\bar{1}.9383$
Ditto $\lambda$ 1,03 . . . . .	=	$\bar{1}.8716$
		$\bar{1}.7651$
	Sum . . .	$= \lambda (a^{10})$

In Table I, $\bar{1}.76$ corresponds to . . .	.8734
In proportional parts ,005 corresponds to	.253
Ditto . . . 0001 corresponds to	.5
	.87604
Consequently $\bar{1}.7651$ corresponds to	

which is the log. of the required value : the number corresponding to this is 7,5169, for the value of the annuity, according to the Carlisle mortality, at 3 per cent. on the joint lives 30 and 40; and by calculation from Mr. MILNE's tables, I find the value should be 7,5168 ; the difference of the two is evidently insignificant. In this way I calculated the log. of the value of the life annuity, at the Carlisle mortality, at 3 per cent. for 10 years, for the joint lives 0 and 10, 10 and 20, 20 and 30, 30 and 40, 40 and 50, 50 and 60, to be ,76580 ; ,90247 ; ,89139 ; ,87604 ; ,86295 ; ,81067 ; and the annuity, or the numbers corresponding to the said logarithms,

5,8318 ; 7,9874 ; 7,7874 ; 7,5169 ; 7,2937 ; 6,4665 ;

and, according to calculation from Mr. MILNE's tables, I get

5,8595 ; 7,992 ; 7,7906 ; 7,5168 ; 7,2916 ; 6,4679.

The difference between the two sets is insignificant, except

perhaps in the values of  $\frac{1}{10} \overbrace{1}^{1.05^{-1}} \underline{0, 10}$ ; that is, the value of the annuity on the joint life of a child just born, with one of the age of 10, at 3 per cent. Had we divided the period in portions, the value might have been obtained as near as we pleased; or we should likewise have obtained greater accuracy, had we assumed an accommodated chance deduced at a more appropriate interest than 5 per cent. See Art. 5, Chap. II.

Example 2. Let it be required to find the value of a life annuity at 3 per cent. for 10 years, at the Carlisle mortality, for the five lives of the age 20, 30, 40, 45 and 50.

In Table VIII. log. of accom. chance for 10 years at age 20 =  $\bar{1}.9685$

Ditto	.	.	.	.	30	=	$\bar{1}.9552$
Ditto	.	.	.	.	40	=	$\bar{1}.9383$
Ditto	.	.	.	.	45	=	$\bar{1}.9367$
Ditto	.	.	.	.	50	=	$\bar{1}.9292$
					$\lambda 1.05^{-10}$	=	$\bar{1}.8716$
					$\lambda (a^{10})$	=	$\bar{1}.5995$

This sought in Table I.; thus,  $\bar{1}.59$  giving ,79035

,009	427
,0005	23
	23

gives ,79485 the  $N^0$  to which log. is 6,2352

for the value of  $\frac{1}{10} \overbrace{1}^{1.03^{-1}} \underline{20, 30, 40, 45, 50}$ .

Example 3. Let it be required to find the value of  $\frac{1}{1} \overbrace{1}^{1.03^{-1}} \underline{b, b+10}$  Carlisle mortality, when  $b = 10$ , that is, for the whole joint lives of 10 and 20. By dividing the whole in portions of ten

years, the operation will stand thus for  $\frac{1}{10} \overbrace{1}^{1.03^{-1}} \underline{b, b+10}$ .

	b=10	b=20	b=30	b=40	b=50	b=60	b=70	b=80	
Log. of accom. ratio } for 10 years = } $\lambda(1,03^{-10}) =$	$\bar{1}.9768$ $\bar{1}.9685$ $\bar{1}.8716$	$\bar{1}.9685$ $\bar{1}.9552$ $\bar{1}.8716$	$\bar{1}.9552$ $\bar{1}.9383$ $\bar{1}.8716$	$\bar{1}.9383$ $\bar{1}.9292$ $\bar{1}.8716$	$\bar{1}.9292$ $\bar{1}.8318$ $\bar{1}.8716$	$\bar{1}.8318$ $\bar{1}.6689$ $\bar{1}.8716$	$\bar{1}.6689$ $\bar{1}.3134$ $\bar{1}.8716$	$\bar{1}.3134$ $\bar{2}.6695$ $\bar{1}.8716$	} from Tab.VIII. Carlisle.
sum . . =	$\bar{1}.8169$	$\bar{1}.7953$	$\bar{1}.7651$	$\bar{1}.7391$	$\bar{1}.6326$	$\bar{1}.3723$	$\bar{2}.8539$	$\bar{3}.8545$	
No <sup>s</sup> corresponding } to sum in Table I. }	.90247	.89139	.87604	.86295	.81067	.69156	.48781	.19146	
Log. of ratios for 10 years = } $\lambda 1,03^{-20}$ . . =	$\bar{1}.97438$ $\bar{1}.96681$ $\bar{1}.87163$	$\bar{1}.94120$ $\bar{1}.92082$ $\bar{1}.74325$	$\bar{1}.89520$ $\bar{1}.85854$ $\bar{1}.61488$	$\bar{1}.83292$ $\bar{1}.77684$ $\bar{1}.48651$	$\bar{1}.75123$ $\bar{1}.59577$ $\bar{1}.35814$	$\bar{1}.57016$ $\bar{1}.19448$ $\bar{1}.22977$	$\bar{1}.16886$ $\bar{2}.36767$ $\bar{1}.10139$		
The log. of the present } worth of each portion }		.70421	.48131	.23157	$\bar{1}.90694$	$\bar{1}.39670$	$\bar{2}.48222$	$\bar{4}.82938$	

And the present worth of each, or the numbers corresponding to the last logarithms are arranged below.

For first 10 years	7.9886	As the method by which the logarithms of the present worth of the different portions are found, may not be seen by every reader, I will explain the operation in the third portion; that is, when the logarithm of the portion first found is anticipated for 20 years. Resume Table VII. log. of real chance for age } 10 living 20 years . . . } $\bar{1}.94120$ Ditto 20 years living . . . } $\bar{1}.92082$ $\lambda(1,03^{-20})$ . . . . . } $\bar{1}.74325$
2nd ditto	5.0607	
3d d <sup>o</sup>	3.0291	
4th d <sup>o</sup>	1.7044	
5th d <sup>o</sup>	.8071	
6th d <sup>o</sup>	.2492	
7th d <sup>o</sup>	.0303	
8th d <sup>o</sup>	.0007	
sum	18.8701	.87604
		.48131

which differs but insignificantly from Mr. MILNE's table, which gives 18.873. In a similar way, I find the value of the joint lives for ages 20 and 30, at 3 per cent. and Carlisle mortality to be 16.745; which, according to Mr. MILNE's table, should be 16.749; which appears to be an insignificant difference.

Example 4. To find, when particular accuracy is not required, according to the formula for the whole of life,

the approximate value of  $\frac{1,03^{-1}}{1} \{ a, a + 10$  at the Carlisle mortality, when  $a = 10, 20, 30,$  &c. call the logarithm of accommodated ratios for an unlimited time at the age  $a, R_a$  standing for the accommodated ratio in Table VI. at the age  $a$ .

$a =$	10	20	30	40	50	60	70	80	90
$R_a$	$\bar{1}.99529$	$\bar{1}.99455$	$\bar{1}.99265$	$\bar{1}.98991$	$\bar{1}.98546$	$\bar{1}.97514$	$\bar{1}.95755$	$\bar{1}.92461$	$\bar{1}.86660$
$R_{a+10}$	$\bar{1}.99455$	$\bar{1}.99265$	$\bar{1}.98991$	$\bar{1}.98546$	$\bar{1}.97514$	$\bar{1}.95755$	$\bar{1}.92461$	$\bar{1}.86660$	$\bar{1}.81282$
$\lambda 1,03^{-1}$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$
	$\bar{1}.97700$	$\bar{1}.97436$	$\bar{1}.96972$	$\bar{1}.96253$	$\bar{1}.94776$	$\bar{1}.91985$	$\bar{1}.86932$	$\bar{1}.77837$	$\bar{1}.66658$
Log. which corresponds to	$\bar{1}.26451$	$\left. \begin{array}{l} 1.20975 \\ .00631 \end{array} \right\}$	$\left. \begin{array}{l} 1.13083 \\ .01062 \end{array} \right\}$	$\left. \begin{array}{l} 1.03886 \\ .00641 \end{array} \right\}$	$\left. \begin{array}{l} .88674 \\ .00667 \end{array} \right\}$	$\left. \begin{array}{l} .68817 \\ .00502 \end{array} \right\}$	$\left. \begin{array}{l} .45337 \\ .00123 \end{array} \right\}$	$\left. \begin{array}{l} .17571 \\ .00093 \end{array} \right\}$	
		$1.21606$	$1.14145$	$1.04527$	$.89341$	$.69319$	$.45460$	$.17664$	
Numbers . .	$18.387$	$16.446$	$13.850$	$11.099$	$7.824$	$4.9339$	$2.8485$	$1.5019$	
Instead of .	$18.873$	$16.749$	$14.449$	$11.954$	$8.729$	$5.565$	$3.229$	$1.589$	

To find the value corresponding to  $\bar{1}.66658$ , not in the table, find the number corresponding complement of the log.  $\bar{1}.6658$ , which number is 2,159; subtract 1, and find the complement of the log. which is  $= \bar{1}.9359165$ , whose number is ,8628. Mr. MILNE's table gives .979. But as it is not always the same rate of interest which gives the best accommodated ratios, in order to try when, for instance, the interest of money is 3 per cent. what rate of interest should be used in determining the ratios, use the following table:\*

Interest.	
1.08	$\lambda (1.08^{-1} \times 1.03) = \bar{1}.979$
1.07	$\lambda (1.07^{-1} \times 1.03) = \bar{1}.983$
1.06	$\lambda (1.06^{-1} \times 1.03) = \bar{1}.987$
1.05	$\lambda (1.05^{-1} \times 1.03) = \bar{1}.991$
1.04	$\lambda (1.04^{-1} \times 1.03) = \bar{1}.996$

} nearly;

\* This is not given as a perfect and unerring rule, but as a method in many cases useful, and which would be perfect for the accommodated ratio of one of the lives, if the other lives followed an exact geometrical ratio throughout; and that the real geometrical ratios were in that case used for them, provided that instead of comparing the said sum with the small table, we take for the base of interest the number whose logarithm is  $-\lambda (1,03)$ , when the interest is 3 per cent.; and it is to be recollected that the methods is only given as a rough approximation.

Add the logarithm of accommodated ratios, as given in the Table VI. of all the lives but one in question, together, and see which of those rates of interest it nearest agrees with, and use that to calculate the life left, and proceed so for

every life ; thus for  $\overset{1,03^{-1}}{1} \left| \begin{array}{l} 30, 40 \end{array} \right.$  ; to find the rate of interest for 30, I observe that  $R_{40} = \bar{1}.9899$  agrees nearest with 6 per cent. in the little table, and  $R_{30} = \bar{1}.99265$  agrees nearest with 5 per cent., I therefore take 6 per cent. for the age 30, and for the other I take 5 per cent. : proceed thus :

Example 5.

R if calculated at 6 per cent.	$\bar{1}.99316$
$R_{40}$ per table . . . . .	$\bar{1}.98991$
$\lambda 1,05^{-1}$ . . . . .	$= \bar{1}.98716$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.97023$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
Proportionate parts . . . . .	$1.14558$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
To which logarithm . . . . .	$1.14885$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
The N <sup>o</sup> corresponding is . . . . .	$14.088$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
Instead of . . . . .	$14.449$

Example 6.

	$\overset{1,03^{-1}}{1} \left  \begin{array}{l} 40, 50 \end{array} \right.$
R at 6 per cent. . . . .	$\bar{1}.99060$
$R_{40}$ at 6 per cent. . . . .	$\bar{1}.98632$
$R_{50}$ at 6 per cent. . . . .	$\bar{1}.98716$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.96408$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.05336$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.00102$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$1.06438$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$11.598$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	Instead of $11.954$

Example 7.

	$\overset{1,03^{-1}}{1} \left  \begin{array}{l} 50, 60 \end{array} \right.$
R at 8 per cent. . . . .	$= \bar{1}.98759$
$R_{50}$ at 6 per cent. . . . .	$= \bar{1}.97599$
$R_{60}$ at 6 per cent. . . . .	$= \bar{1}.98716$
$\lambda 1,03^{-1}$ . . . . .	$= \bar{1}.98716$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.95074$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.91357$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.00687$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.92044$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
which log. corresponds . . . . .	$.8318$
instead of . . . . .	$.8729$



I observe that I have not given any table of the logarithm of the accommodated ratios for an unlimited term, except that calculated with 5 per cent. as a radix ; but by the assistance of a table of life annuities, for single life at different rates per cent., this will enable us, independent of certain exceptions, to derive the quantity for the same rates per cent. for any radix at the per cent. contained in the second table ; thus to find  $R$  Carlisle mortality, radix 8 per cent. I look to the Carlisle table of single lives at 8 per cent., and I find the value of the annuity on the life of 50 = 8.987, I search the age to which this will correspond at 5 per cent. and I find sufficiently nearly 59,82 for the age corresponding, to which from my table (with the radix at 5 per cent.) for the log. of ratios I find  $\bar{1}.97536$  ; to this I add log. of  $\frac{1.08}{1.05}$  ; that is, ,01223, and we get  $\bar{1}.98759$ , the same as given on the other side. This method is accurately consistent with the definition of accommodated ratios for unlimited periods ; and if this description of accommodated ratios at a certain rate per cent. be given for one table, for which at the same rate per cent. we have the value of single lives, we may find the same description of accommodated ratios for any other table of mortality for which, at the same rate per cent. we have a table of the value of single lives : thus, suppose the logarithm of this description of accommodated ratios be given for the Carlisle table at five per cent., and the same be required for

the Northampton for the age 60, at the same rate ;  $\frac{1.05}{1} \bar{1} \left| \begin{matrix} -1 \\ 60 \end{matrix} \right.$   
 Northampton = 8,392, this being sought in the Carlisle

table for  $1.05^{-1} \int_1^x$  gives  $x = 62,41$  for the corresponding age; seek the logarithm of accommodated ratios for an unlimited term, corresponding to this for Carlisle, for the age 6,241, and we have  $\bar{1}.9723$ , agreeing with the table given.

Previously to concluding this chapter, I shall add a small table, which will be found very useful in the application of the methods here proposed.

$n$	Log. of $1,03^{-n}$	Log. of $1,035^{-n}$	Log. of $1,04^{-n}$	Log. of $1,045^{-n}$	Log. of $1,05^{-n}$
1	$\bar{1}.9871628$	$\bar{1}.9850597$	$\bar{1}.9829667$	$\bar{1}.9808837$	$\bar{1}.9788107$
2	$\bar{1}.9743256$	$\bar{1}.9701193$	$\bar{1}.9659333$	$\bar{1}.9617674$	$\bar{1}.9576214$
3	$\bar{1}.9614883$	$\bar{1}.9551790$	$\bar{1}.9489000$	$\bar{1}.9426511$	$\bar{1}.9364321$
4	$\bar{1}.9486511$	$\bar{1}.9402386$	$\bar{1}.9318666$	$\bar{1}.9235348$	$\bar{1}.9152428$
5	$\bar{1}.9358139$	$\bar{1}.9252983$	$\bar{1}.9148333$	$\bar{1}.9044185$	$\bar{1}.8940535$
6	$\bar{1}.9229767$	$\bar{1}.9103579$	$\bar{1}.8978000$	$\bar{1}.8853023$	$\bar{1}.8728642$
7	$\bar{1}.9101394$	$\bar{1}.8954176$	$\bar{1}.8807666$	$\bar{1}.8661860$	$\bar{1}.8516749$
8	$\bar{1}.8973022$	$\bar{1}.8804772$	$\bar{1}.8637333$	$\bar{1}.8470697$	$\bar{1}.8304856$
9	$\bar{1}.8844650$	$\bar{1}.8655369$	$\bar{1}.8466999$	$\bar{1}.8279534$	$\bar{1}.8092963$
10	$\bar{1}.8716278$	$\bar{1}.8505965$	$\bar{1}.8296666$	$\bar{1}.8088371$	$\bar{1}.7881070$

General Table I.  $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^i-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^i-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^i-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.00$	,00163 ,00199,6	,00	0200 1796	0399 1597	0599 1397	0798 1198	0998	$\bar{3}.25$	,05295 ,00211,7	,00	0212 1905	0423 1694	0635 1482	0847 1270	1059
$\bar{3}.01$	,0036,2 ,00200,3		0200 1803	0401 1602	0601 1402	0801 1202	1002	$\bar{3}.26$	,05506 ,00212,2		0212 1910	0424 1698	0637 1485	0849 1273	1061
$\bar{3}.02$	,00563 ,00200,6		0201 1805	0401 1605	0602 1404	0802 1204	1003	$\bar{3}.27$	,05719 ,00212,7		0213 1914	0425 1702	0638 1489	0851 1276	1064
$\bar{3}.03$	,00763 ,00201,0		0201 1809	0402 1608	0603 1407	0804 1206	1005	$\bar{3}.28$	,05931 ,00213,2		0213 1919	0426 1706	0640 1492	0853 1279	1066
$\bar{3}.04$	,00964 ,00201,5		0202 1814	0403 1612	0605 1411	0806 1209	1008	$\bar{3}.29$	,06144 ,00213,7		0214 1923	0427 1710	0641 1496	0855 1282	1069
$\bar{3}.05$	,01166 ,00202,0		0202 1818	0404 1616	0606 1414	0808 1212	1010	$\bar{3}.30$	,06358 ,00214,3		0214 1929	0429 1714	0643 1500	0857 1286	1072
$\bar{3}.06$	,01368 ,00202,4		0202 1822	0405 1619	0607 1417	0810 1214	1012	$\bar{3}.31$	,06572 ,00214,8		0215 1933	0430 1718	0644 1504	0859 1289	1074
$\bar{3}.07$	,01570 ,00202,8		0203 1825	0406 1622	0608 1420	0811 1217	1014	$\bar{3}.32$	,06787 ,00215,3		0215 1938	0431 1722	0646 1507	0861 1292	1077
$\bar{3}.08$	,01773 ,00203,4		0203 1831	0407 1627	0610 1424	0814 1220	1017	$\bar{3}.33$	,07002 ,00216,0		0216 1944	0432 1728	0648 1512	0864 1296	1080
$\bar{3}.09$	,01976 ,00203,8		0204 1834	0408 1630	0611 1427	0815 1223	1019	$\bar{3}.34$	,07218 ,00216,4		0216 1948	0433 1731	0649 1515	0866 1298	1082
$\bar{3}.10$	,02180 ,00204,3		0204 1839	0409 1634	0613 1430	0817 1226	1022	$\bar{3}.35$	,07435 ,00217,0		0217 1953	0434 1736	0651 1519	0868 1302	1085
$\bar{3}.11$	,02384 ,00204,7		0205 1842	0409 1638	0614 1433	0819 1228	1024	$\bar{3}.36$	,07652 ,00217,5		0218 1958	0435 1740	0653 1523	0870 1305	1089
$\bar{3}.12$	,02589 ,00205,2		0205 1847	0410 1642	0616 1436	0821 1231	1026	$\bar{3}.37$	,07869 ,00218,0		0218 1962	0436 1744	0654 1526	0872 1308	1090
$\bar{3}.13$	,02794 ,00205,7		0206 1851	0411 1646	0617 1440	0823 1234	1029	$\bar{3}.38$	,08087 ,00218,7		0219 1968	0437 1750	0656 1531	0875 1312	1094
$\bar{3}.14$	,03000 ,00206,1		0206 1855	0412 1649	0618 1443	0824 1237	1031	$\bar{3}.39$	,08306 ,00219,2		0219 1973	0438 1754	0658 1534	0877 1315	1096
$\bar{3}.15$	,03206 ,00206,5		0207 1859	0413 1654	0620 1446	0826 1239	1033	$\bar{3}.40$	,08525 ,00219,7		0220 1977	0439 1758	0659 1538	0879 1318	1099
$\bar{3}.16$	,03407 ,00207,3		0207 1866	0415 1658	0622 1451	0829 1244	1037	$\bar{3}.41$	,08745 ,00220,4		0220 1984	0441 1763	0661 1543	0882 1322	1102
$\bar{3}.17$	,03620 ,00207,6		0208 1868	0415 1661	0623 1453	0830 1246	1038	$\bar{3}.42$	,08965 ,00221,0		0221 1989	0442 1768	0663 1547	0884 1326	1105
$\bar{3}.18$	,03827 ,00208,1		0208 1873	0416 1665	0624 1457	0832 1249	1041	$\bar{3}.43$	,09186 ,00221,4		0221 1993	0443 1771	0664 1551	0886 1328	1107
$\bar{3}.19$	,04036 ,00208,6		0209 1877	0417 1669	0626 1460	0834 1252	1043	$\bar{3}.44$	,09408 ,00222,1		0222 1999	0444 1777	0666 1555	0888 1333	1111
$\bar{3}.20$	,04244 ,00209,1		0209 1882	0418 1673	0627 1464	0836 1255	1046	$\bar{3}.45$	,09630 ,00222,6		0223 2003	0445 1781	0668 1558	0890 1336	1113
$\bar{3}.21$	,04453 ,00209,6		0210 1886	0419 1677	0629 1467	0838 1258	1048	$\bar{3}.46$	,09852 ,00223,2		0223 2009	0446 1786	0670 1562	0893 1339	1116
$\bar{3}.22$	,04663 ,00210,1		0210 1891	0420 1681	0630 1471	0840 1261	1051	$\bar{3}.47$	,10076 ,00223,8		0224 2014	0448 1790	0671 1567	0895 1343	1119
$\bar{3}.23$	,04873 ,00210,6		0211 1895	0421 1685	0632 1474	0842 1264	1053	$\bar{3}.48$	,10300 ,00224,4		0224 2020	0449 1795	0673 1571	0898 1346	1122
$\bar{3}.24$	,05084 ,00211,1		0211 1900	0422 1689	0633 1478	0844 1267	1056	$\bar{3}.49$	,10524 ,00225,0		0225 2025	0450 1800	0675 1575	0900 1350	1125

General Table I.  $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5		
$\bar{3}.50$	,10749 ,00225,6	,00	0226 2030	0451 1805	0677 1579	0902 1354	1128	$\bar{3}.75$	,16581 ,00242,0	,00	0242 2178	0484 1936	0726 1694	0968 1452	1210
$\bar{3}.51$	,10975 ,00226,2		0226 2036	0452 1810	0679 1583	0905 1357	1131	$\bar{3}.76$	,16823 ,00242,7		0243 2184	0485 1942	0728 1699	0971 1456	1214
$\bar{3}.52$	,11201 ,00226,8		0227 2041	0454 1814	0680 1588	0907 1361	1134	$\bar{3}.77$	,17065 ,00243,5		0244 2192	0487 1948	0731 1705	0974 1461	1218
$\bar{3}.53$	,11428 ,00227,5		0228 2048	0455 1820	0683 1593	0910 1365	1138	$\bar{3}.78$	,17309 ,00244,2		0244 2198	0488 1954	0733 1709	0977 1465	1221
$\bar{3}.54$	,11655 ,00228,1		0228 2053	0456 1825	0684 1597	0912 1369	1141	$\bar{3}.79$	,17553 ,00244,9		0245 2204	0490 1959	0735 1714	0980 1469	1225
$\bar{3}.55$	,11883 ,00228,7		0229 2058	0457 1830	0686 1601	0915 1372	1144	$\bar{3}.80$	,17798 ,00245,6		0246 2210	0491 1965	0737 1719	0982 1474	1228
$\bar{3}.56$	,12112 ,00229,2		0229 2063	0458 1834	0688 1604	0917 1375	1146	$\bar{3}.81$	,18044 ,00246,4		0246 2218	0493 1971	0739 1725	0986 1478	1232
$\bar{3}.57$	,12341 ,00230,1		0230 2071	0460 1841	0690 1611	0920 1381	1151	$\bar{3}.82$	,18290 ,00247,1		0247 2224	0494 1977	0741 1730	0988 1482	1236
$\bar{3}.58$	,12571 ,00230,6		0231 2075	0461 1845	0692 1614	0922 1384	1153	$\bar{3}.83$	,18537 ,00247,8		0248 2230	0496 1982	0743 1735	0991 1487	1239
$\bar{3}.59$	,12802 ,00231,2		0231 2081	0462 1850	0694 1618	0925 1387	1156	$\bar{3}.84$	,18785 ,00248,6		0249 2237	0497 1989	0746 1740	0994 1492	1243
$\bar{3}.60$	,13033 ,00231,9		0232 2087	0464 1855	0696 1623	0928 1391	1160	$\bar{3}.85$	,19034 ,00249,3		0249 2244	0499 1994	0748 1745	0997 1496	1247
$\bar{3}.61$	,13265 ,00232,5		0233 2093	0465 1860	0698 1628	0930 1395	1163	$\bar{3}.86$	,19284 ,00250,1		0250 2251	0500 2001	0750 1751	1000 1501	1251
$\bar{3}.62$	,13497 ,00233,0		0233 2097	0466 1864	0699 1631	0932 1398	1165	$\bar{3}.87$	,19533 ,00250,9		0251 2258	0502 2007	0753 1756	1004 1505	1255
$\bar{3}.63$	,13730 ,00233,8		0234 2104	0468 1870	0701 1637	0935 1403	1169	$\bar{3}.88$	,19784 ,00251,6		0252 2264	0503 2013	0755 1761	1006 1510	1258
$\bar{3}.64$	,13964 ,00234,5		0235 2111	0469 1876	0704 1642	0938 1407	1174	$\bar{3}.89$	,20035 ,00252,4		0252 2272	0505 2019	0757 1767	1010 1514	1262
$\bar{3}.65$	,14199 ,00235,1		0235 2116	0470 1881	0705 1646	0940 1411	1176	$\bar{3}.90$	,20288 ,00253,2		0253 2279	0506 2026	0760 1772	1013 1519	1266
$\bar{3}.66$	,14434 ,00235,8		0236 2122	0472 1886	0707 1651	0943 1414	1179	$\bar{3}.91$	,20541 ,00254,1		0254 2287	0508 2033	0762 1779	1016 1525	1271
$\bar{3}.67$	,14670 ,00236,6		0237 2129	0473 1893	0710 1656	0946 1420	1183	$\bar{3}.92$	,20795 ,00254,7		0255 2292	0509 2038	0764 1783	1019 1528	1274
$\bar{3}.68$	,14906 ,00237,1		0237 2134	0474 1897	0711 1660	0948 1423	1186	$\bar{3}.93$	,21050 ,00255,7		0256 2301	0511 2046	0767 1790	1023 1534	1279
$\bar{3}.69$	,15143 ,00237,9		0238 2141	0476 1903	0714 1665	0952 1427	1190	$\bar{3}.94$	,21306 ,00256,3		0256 2307	0513 2050	0769 1794	1025 1538	1282
$\bar{3}.70$	,15381 ,00238,5		0239 2147	0477 1908	0716 1670	0954 1431	1193	$\bar{3}.95$	,21562 ,00257,2		0257 2315	0514 2058	0772 1800	1029 1543	1286
$\bar{3}.71$	,15620 ,00239,2		0239 2153	0478 1914	0718 1674	0957 1435	1196	$\bar{3}.96$	,21819 ,00258,0		0258 2322	0516 2064	0774 1806	1032 1548	1290
$\bar{3}.72$	,15859 ,00239,9		0240 2159	0480 1919	0720 1679	0960 1439	1200	$\bar{3}.97$	,22077 ,00258,8		0259 2329	0518 2070	0776 1812	1035 1553	1294
$\bar{3}.73$	,16099 ,00240,6		0241 2165	0481 1925	0722 1684	0962 1444	1203	$\bar{3}.98$	,22336 ,00259,6		0260 2336	0519 2077	0779 1817	1038 1558	1298
$\bar{3}.74$	,16339 ,00241,3		0241 2172	0483 1930	0724 1689	0965 1448	1207	$\bar{3}.99$	,22559 ,00260,4		0260 2344	0521 2083	0781 1823	1042 1562	1302

General Table I.  $\lambda (a^{10}), \lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$ .

$\lambda (a^{10})$	$\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$	1 9	2 8	3 7	4 6	5	$\lambda (a^{10})$	$\lambda \left( \frac{1-a^{10}}{a^{-1}-1} \right)$	1 9	2 8	3 7	4 6	5		
$\bar{z}.00$	,22856 ,00261,4	,00	0261 2353	0523 2091	0784 1830	1046 1568	1307	$\bar{z}.25$	,29652 ,00283,9	,00	0284 2555	0568 2271	0852 1987	1136 1703	1420
$\bar{z}.01$	,23117 ,00262,0		0262 2358	0524 2096	0786 1834	1048 1572	1310	$\bar{z}.26$	,29936 ,00284,9		0285 2564	0570 2279	0855 1994	1140 1709	1425
$\bar{z}.02$	,23379 ,00263,0		0263 2367	0526 2104	0789 1841	1052 1578	1315	$\bar{z}.27$	,30221 ,00285,9		0286 2573	0572 2287	0858 2001	1144 1715	1430
$\bar{z}.03$	,23642 ,00263,8		0264 2374	0528 2110	0791 1847	1055 1583	1319	$\bar{z}.28$	,30507 ,00286,8		0287 2581	0574 2294	0860 2008	1147 1721	1434
$\bar{z}.04$	,23906 ,00264,7		0265 2382	0529 2118	0794 1853	1059 1588	1324	$\bar{z}.29$	,30794 ,00287,9		0288 2591	0576 2303	0864 2015	1152 1727	1440
$\bar{z}.05$	,24171 ,00265,6		0266 2390	0531 2125	0797 1859	1062 1594	1328	$\bar{z}.30$	,31081 ,00288,9		0289 2600	0578 2311	0867 2022	1156 1733	1445
$\bar{z}.06$	,24036 ,00266,3		0266 2397	0533 2130	0799 1864	1065 1598	1332	$\bar{z}.31$	,31370 ,00289,9		0290 2609	0580 2319	0870 2029	1160 1739	1450
$\bar{z}.07$	,24703 ,00267,0		0267 2403	0534 2136	0801 1869	1068 1602	1335	$\bar{z}.32$	,31660 ,00290,9		0291 2618	0582 2327	0873 2036	1164 1745	1455
$\bar{z}.08$	,24970 ,00268,5		0269 2417	0537 2148	0806 1880	1074 1611	1343	$\bar{z}.33$	,31951 ,00291,9		0292 2627	0584 2335	0876 2043	1168 1751	1460
$\bar{z}.09$	,25238 ,00269,0		0269 2421	0538 2152	0807 1883	1076 1614	1345	$\bar{z}.34$	,32243 ,00293,0		0293 2637	0586 2344	0879 2051	1172 1758	1465
$\bar{z}.10$	,25507 ,00269,9		0270 2429	0540 2159	0810 1889	1080 1619	1350	$\bar{z}.35$	,32536 ,00294,0		0294 2646	0588 2352	0882 2058	1176 1764	1470
$\bar{z}.11$	,25777 ,00270,8		0271 2437	0542 2166	0812 1896	1083 1625	1354	$\bar{z}.36$	,32830 ,00295,0		0295 2655	0590 2360	0885 2065	1180 1770	1475
$\bar{z}.12$	,26048 ,00271,7		0272 2445	0543 2174	0815 1902	1087 1630	1359	$\bar{z}.37$	,33125 ,00296,1		0296 2665	0592 2369	0888 2073	1184 1777	1481
$\bar{z}.13$	,26320 ,00272,6		0273 2453	0545 2181	0818 1908	1090 1636	1363	$\bar{z}.38$	,33421 ,00297,1		0297 2674	0594 2377	0891 2080	1188 1783	1485
$\bar{z}.14$	,26592 ,00273,5		0274 2462	0547 2188	0821 1915	1094 1641	1368	$\bar{z}.39$	,33718 ,00298,2		0298 2684	0596 2386	0895 2087	1193 1789	1491
$\bar{z}.15$	,26866 ,00274,6		0275 2471	0549 2197	0824 1922	1098 1648	1373	$\bar{z}.40$	,34016 ,00299,3		0299 2694	0599 2394	0898 2095	1197 1796	1497
$\bar{z}.16$	,27140 ,00275,2		0275 2477	0550 2202	0826 1926	1101 1651	1376	$\bar{z}.41$	,34316 ,00300,3		0300 2703	0601 2402	0901 2102	1201 1802	1502
$\bar{z}.17$	,27415 ,00276,3		0276 2487	0553 2210	0829 1934	1105 1658	1382	$\bar{z}.42$	,34616 ,00301,4		0301 2713	0603 2411	0904 2110	1206 1808	1507
$\bar{z}.18$	,27692 ,00277,2		0277 2495	0554 2218	0832 1940	1109 1663	1386	$\bar{z}.43$	,34917 ,00302,5		0303 2723	0605 2420	0908 2118	1210 1815	1513
$\bar{z}.19$	,27969 ,00278,1		0278 2503	0556 2225	0834 1947	1112 1669	1391	$\bar{z}.44$	,35220 ,00303,6		0304 2732	0607 2429	0911 2125	1214 1822	1518
$\bar{z}.20$	,28247 ,00279,1		0279 2512	0558 2233	0837 1954	1116 1675	1396	$\bar{z}.45$	,35523 ,00304,7		0305 2742	0609 2438	0914 2133	1219 1828	1524
$\bar{z}.21$	,28526 ,00280,1		0280 2521	0560 2241	0840 1961	1120 1681	1401	$\bar{z}.46$	,35828 ,00305,8		0306 2752	0612 2446	0917 2141	1223 1835	1529
$\bar{z}.22$	,28806 ,00281,1		0281 2530	0562 2249	0843 1968	1124 1687	1406	$\bar{z}.47$	,36134 ,00306,9		0307 2762	0614 2455	0921 2148	1228 1841	1535
$\bar{z}.23$	,29087 ,00281,9		0282 2537	0564 2255	0846 1973	1128 1691	1410	$\bar{z}.48$	,36441 ,00308,1		0308 2773	0616 2465	0924 2157	1232 1849	1541
$\bar{z}.24$	,29369 ,00282,9		0283 2546	0566 2263	0849 1980	1132 1697	1415	$\bar{z}.49$	,36749 ,00309,2		0309 2783	0618 2474	0928 2164	1237 1855	1546

General Table I.  $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5		
$\bar{2}.50$	,37058 ,00310,3 ,37368	,00	0310 2794	0621 2482	0931 2172	1241 1862	1552	$\bar{2}.75$	,45174 ,00340,8	,00	0341 3067	0682 2726	1022 2386	1363 2045	1704
$\bar{2}.51$	,00311,5 ,37680		0312 2804	0623 2492	0935 2181	1246 1869	1558	$\bar{2}.76$	,45515 ,00342,3		0342 3081	0685 2738	1027 2396	1369 2054	1712
$\bar{2}.52$	,00312,6 ,37993		0313 2813	0625 2501	0938 2188	1250 1876	1563	$\bar{2}.77$	,45857 ,00343,5		0343 3092	0687 2748	1031 2405	1374 2061	1718
$\bar{2}.53$	,00313,8 ,38306		0314 2824	0628 2510	0941 2197	1255 1883	1569	$\bar{2}.78$	,46201 ,00345,0		0345 3105	0690 2760	1035 2415	1380 2070	1725
$\bar{2}.54$	,00314,9 ,38621		0315 2834	0630 2519	0945 2204	1260 1889	1575	$\bar{2}.79$	,46546 ,00346,1		0346 3115	0692 2769	1038 2423	1384 2077	1731
$\bar{2}.55$	,00316,1 ,38937		0316 2845	0632 2529	0948 2213	1265 1897	1581	$\bar{2}.80$	,46892 ,00347,5		0348 3128	0695 2780	1043 2433	1390 2085	1738
$\bar{2}.56$	,00317,3 ,39255		0317 2856	0635 2538	0952 2221	1269 1904	1587	$\bar{2}.81$	,47239 ,00348,9		0349 3140	0698 2791	1047 2442	1396 2093	1745
$\bar{2}.57$	,00318,5 ,39573		0319 2867	0637 2548	0956 2230	1274 1911	1593	$\bar{2}.82$	,47588 ,00350,2		0350 3152	0700 2802	1051 2451	1410 2101	1751
$\bar{2}.58$	,00319,6 ,39893		0320 2876	0639 2557	0959 2237	1278 1918	1598	$\bar{2}.83$	,47938 ,00351,6		0352 3164	0703 2813	1055 2461	1406 2110	1758
$\bar{2}.59$	,00320,8 ,40213		0321 2887	0642 2566	0962 2246	1283 1925	1604	$\bar{2}.84$	,48290 ,00353,0		0353 3177	0706 2824	1059 2471	1412 2118	1765
$\bar{2}.60$	,00322,0 ,40536		0322 2910	0644 2586	0966 2263	1288 1940	1610	$\bar{2}.85$	,48643 ,00354,3		0354 3189	0709 2834	1063 2486	1417 2126	1772
$\bar{2}.61$	,00323,3 ,40859		0323 2921	0647 2596	0970 2272	1293 1947	1617	$\bar{2}.86$	,48997 ,00355,8		0356 3202	0712 2846	1067 2491	1423 2135	1779
$\bar{2}.62$	,00324,5 ,41183		0325 2932	0649 2605	0974 2279	1298 1954	1623	$\bar{2}.87$	,49353 ,00357,1		0357 3214	0714 2857	1071 2500	1428 2143	1786
$\bar{2}.63$	,00325,6 ,41509		0326 2941	0651 2614	0977 2288	1302 1961	1628	$\bar{2}.88$	,49710 ,00358,5		0359 3227	0717 2868	1076 2510	1434 2151	1793
$\bar{2}.64$	,00326,8 ,41836		0327 2955	0654 2626	0980 2298	1307 1970	1634	$\bar{2}.89$	,50069 ,00360,0		0360 3240	0720 2880	1080 2520	1440 2160	1800
$\bar{2}.65$	,00328,3 ,42164		0328 2965	0657 2635	0985 2306	1313 1976	1642	$\bar{2}.90$	,50429 ,00361,4		0361 3253	0723 2891	1084 2530	1446 2168	1807
$\bar{2}.66$	,00329,4 ,42493		0329 2975	0659 2645	0988 2314	1318 1984	1647	$\bar{2}.91$	,50790 ,00362,7		0363 3264	0725 2902	1088 2539	1451 2176	1814
$\bar{2}.67$	,00330,6 ,42833		0331 2987	0661 2655	0992 2323	1322 1991	1653	$\bar{2}.92$	,51153 ,00364,2		0364 3278	0728 2914	1093 2549	1457 2185	1821
$\bar{2}.68$	,00331,9 ,43156		0332 2999	0664 2666	0996 2332	1328 1999	1660	$\bar{2}.93$	,51517 ,00365,6		0366 3290	0731 2925	1097 2559	1462 2194	1828
$\bar{2}.69$	,00333,2 ,43490		0333 3010	0666 2675	1000 2341	1333 2006	1666	$\bar{2}.94$	,51883 ,00367,1		0367 3304	0734 2937	1101 2570	1468 2203	1836
$\bar{2}.70$	,00334,4 ,43824		0334 3021	0669 2686	1003 2350	1338 2014	1672	$\bar{2}.95$	,52250 ,00368,5		0369 3317	0737 2948	1106 2580	1474 2211	1843
$\bar{2}.71$	,00335,7 ,44159		0336 3034	0671 2697	1007 2360	1343 2023	1679	$\bar{2}.96$	,52618 ,00369,9		0370 3329	0740 2959	1110 2589	1480 2219	1850
$\bar{2}.72$	,00337,1 ,44496		0337 3044	0674 2706	1011 2367	1348 2029	1686	$\bar{2}.97$	,52988 ,00371,4		0371 3343	0743 2971	1114 2600	1486 2228	1857
$\bar{2}.73$	,00338,2 ,44835		0338 3056	0676 2717	1015 2377	1353 2038	1691	$\bar{2}.98$	,53360 ,00372,9		0373 3356	0746 2983	1119 2610	1492 2237	1865
$\bar{2}.74$	,00339,6		0340	0679	1019	1358	1698	$\bar{2}.99$	,53732 ,00374,4		0374	0749	1123	1498	1872

General Table I.  $\lambda (a^{10}), \lambda \left(\frac{1-a^{10}}{a^1-1}\right)$ .

$\lambda (a^{10})$	$\lambda \left(\frac{1-a^{10}}{a^1-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda (a^{10})$	$\lambda \left(\frac{1-a^{10}}{a^1-1}\right)$		1 9	2 8	3 7	4 6	5
1.00	,54107 ,00375,8	,00	0376 3382	0752 3006	1127 2631	1503 2255	1879	1.25	,63964 ,00415,3	,00	0415 3748	0831 3322	1246 2907	1661 2492	2077
1.01	,54483 ,00377,4		0377 3397	0755 3019	1132 2642	1510 2264	1887	1.26	,64380 ,00416,7		0417 3750	0833 3334	1250 2917	1667 2500	2084
1.02	,54860 ,00378,7		0379 3408	0757 3030	1136 2651	1515 2273	1894	1.27	,64796 ,00418,4		0418 3766	0837 3347	1255 2929	1674 2510	2092
1.03	,55239 ,00380,3		0380 3423	0761 3042	1141 2662	1521 2282	1902	1.28	,65215 ,00420,0		0420 3780	0840 3360	1260 2940	1680 2520	2100
1.04	,55619 ,00381,9		0382 3437	0764 3055	1146 2673	1528 2291	1910	1.29	,65635 ,00421,7		0422 3795	0843 3374	1265 2952	1687 2530	2109
1.05	,56001 ,00383,3		0383 3450	0767 3066	1150 2683	1533 2300	1917	1.30	,66056 ,00423,3		0423 3810	0847 3386	1270 2963	1693 2540	2117
1.06	,56384 ,00384,9		0385 3464	0770 3079	1155 2694	1540 2309	1925	1.31	,66480 ,00425,0		0425 3825	0850 3400	1275 2975	1700 2550	2125
1.07	,56769 ,00386,4		0386 3478	0773 3091	1159 2705	1546 2318	1932	1.32	,66905 ,00426,7		0427 3840	0853 3414	1280 2987	1707 2560	2134
1.08	,57156 ,00388,0		0388 3492	0776 3104	1164 2716	1552 2328	1940	1.33	,67331 ,00428,4		0428 3856	0857 3427	1285 2999	1714 2570	2143
1.09	,57544 ,00389,4		0389 3505	0779 3115	1168 2726	1558 2336	1947	1.34	,67760 ,00430,1		0430 3871	0860 3441	1290 3011	1720 2581	2151
1.10	,57933 ,00391,0		0391 3519	0782 3128	1173 2737	1564 2346	1955	1.35	,68190 ,00431,7		0432 3885	0863 3454	1295 3022	1727 2590	2159
1.11	,58324 ,00392,5		0393 3533	0785 3140	1178 2748	1570 2355	1963	1.36	,68622 ,00434,0		0434 3906	0868 3472	1302 3038	1736 2604	2170
1.12	,58717 ,00394,2		0394 3548	0788 3154	1183 2759	1577 2365	1971	1.37	,69056 ,00434,7		0435 3912	0869 3478	1304 3043	1739 2608	2174
1.13	,59111 ,00395,9		0396 3563	0792 3167	1188 2771	1584 2375	1980	1.38	,69490 ,00437,0		0437 3933	0874 3496	1311 3059	1748 2622	2185
1.14	,59507 ,00397,1		0397 3574	0794 3177	1191 2780	1588 2383	1986	1.39	,69927 ,00438,7		0439 3948	0877 3510	1316 3071	1755 2632	2194
1.15	,59904 ,00398,8		0399 3589	0798 3190	1196 2792	1595 2393	1994	1.40	,70366 ,00440,4		0440 3964	0881 3523	1321 3083	1762 2642	2202
1.16	,60303 ,00400,5		0401 3605	0801 3204	1202 2804	1602 2403	2003	1.41	,70806 ,00442,1		0442 3979	0884 3537	1326 3095	1768 2653	2211
1.17	,60703 ,00402,1		0402 3619	0804 3217	1206 2815	1608 2413	2011	1.42	,71249 ,00443,8		0444 3994	0888 3550	1331 3107	1775 2663	2219
1.18	,61105 ,00403,7		0404 3633	0827 3230	1211 2826	1615 2422	2019	1.43	,71692 ,00445,5		0445 4005	0890 3560	1335 3115	1780 2670	2225
1.19	,61509 ,00405,3		0405 3648	0811 3242	1216 2837	1621 2432	2027	1.44	,72137 ,00448,0		0448 4032	0896 3584	1344 3136	1792 2688	2240
1.20	,61914 ,00406,9		0407 3662	0814 3255	1221 2848	1628 2441	2035	1.45	,72585 ,00449,1		0449 4042	0898 3593	1347 3144	1796 2695	2246
1.21	,62321 ,00408,5		0409 3677	0817 3268	1226 2860	1634 2451	2043	1.46	,73035 ,00450,8		0451 4057	0902 3606	1352 3156	1803 2705	2254
1.22	,62729 ,00410,1		0410 3691	0820 3281	1230 2871	1640 2461	2051	1.47	,73485 ,00452,7		0453 4074	0905 3622	1358 3169	1811 2716	2264
1.23	,63140 ,00411,8		0412 3706	0824 3294	1235 2883	1647 2471	2059	1.48	,73938 ,00454,4		0454 4090	0909 3635	1363 3181	1818 2726	2272
1.24	,63551 ,00413,1		0413 3718	0826 3305	1239 2892	1652 2489	2066	1.49	,74393 ,00456,1		0456 4105	0912 3649	1368 3193	1824 2737	2281

General Table I.  $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a-1}\right)$ .

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{1}.50$	,74849 ,00458,0	,00	0458 4122	0916 3664	1374 3206	1832 2748	2290	$\bar{1}.75$	,86842 ,00503,7	,00	0504 4533	1007 4030	1511 3526	2015 3022	2519
$\bar{1}.51$	,75307 ,00459,8		0460 4138	0920 3678	1379 3219	1839 2759	2299	$\bar{1}.76$	,87346 ,00505,6		0506 4550	1011 4045	1517 3539	2022 3034	2528
$\bar{1}.52$	,75766 ,00461,6		0462 4154	0923 3693	1385 3231	1846 2770	2308	$\bar{1}.77$	,87851 ,00507,4		0507 4507	1015 4059	1522 3552	2030 3044	2537
$\bar{1}.53$	,76228 ,00463,3		0463 4170	0927 3706	1390 3243	1853 2780	2317	$\bar{1}.78$	,88359 ,00509,4		0509 4585	1019 4075	1528 3566	2038 3056	2547
$\bar{1}.54$	,76691 ,00465,2		0465 4187	0930 3722	1396 3256	1861 2791	2326	$\bar{1}.79$	,88868 ,00511,2		0511 4611	1022 4090	1534 3578	2045 3067	2556
$\bar{1}.55$	,77156 ,00466,7		0467 4200	0933 3734	1400 3267	1867 2800	2334	$\bar{1}.80$	,89379 ,00513,1		0513 4618	1026 4105	1539 3592	2052 3079	2566
$\bar{1}.56$	,77623 ,00469,0		0469 4221	0938 3752	1407 3283	1876 2814	2345	$\bar{1}.81$	,89892 ,00515,0		0515 4635	1030 4120	1545 3605	2060 3090	2575
$\bar{1}.57$	,78092 ,00470,4		0470 4234	0941 3763	1411 3293	1882 2822	2352	$\bar{1}.82$	,90407 ,00516,8		0517 4651	1034 4134	1550 3618	2067 3101	2584
$\bar{1}.58$	,78562 ,00472,4		0472 4252	0945 3779	1417 3307	1890 2834	2362	$\bar{1}.83$	,90924 ,00518,8		0519 4669	1038 4150	1556 3632	2075 3113	2594
$\bar{1}.59$	,79035 ,00474,1		0474 4267	0948 3793	1422 3319	1896 2845	2371	$\bar{1}.84$	,91443 ,00520,6		0521 4685	1041 4165	1562 3644	2082 3124	2603
$\bar{1}.60$	,79509 ,00476,0		0476 4284	0952 3808	1428 3332	1904 2856	2380	$\bar{1}.85$	,91964 ,00522,6		0523 4703	1045 4181	1568 3658	2090 3136	2613
$\bar{1}.61$	,79985 ,00477,8		0478 4300	0956 3832	1433 3345	1911 2867	2389	$\bar{1}.86$	,92486 ,00524,4		0524 4720	1049 4195	1573 3671	2098 3146	2622
$\bar{1}.62$	,80463 ,00479,6		0480 4316	0959 3837	1439 3357	1918 2878	2398	$\bar{1}.87$	,93011 ,00526,1		0526 4735	1052 4209	1578 3683	2104 3157	2631
$\bar{1}.63$	,80942 ,00481,5		0482 4334	0963 3852	1445 3371	1926 2889	2408	$\bar{1}.88$	,93537 ,00528,3		0528 4755	1057 4226	1585 3698	2113 3170	2642
$\bar{1}.64$	,81424 ,00483,3		0483 4350	0967 3866	1450 3383	1933 2900	2417	$\bar{1}.89$	,94065 ,00530,0		0530 4770	1060 4240	1590 3710	2120 3180	2650
$\bar{1}.65$	,81907 ,00485,7		0486 4371	0971 3886	1457 3400	1943 2914	2429	$\bar{1}.90$	,94595 ,00532,0		0532 4788	1064 4256	1596 3724	2128 3192	2660
$\bar{1}.66$	,82393 ,00486,4		0486 4378	0973 3891	1459 3405	1946 2918	2432	$\bar{1}.91$	,95127 ,00533,8		0534 4804	1068 4270	1601 3737	2135 3203	2669
$\bar{1}.67$	,82879 ,00488,8		0489 4399	0978 3910	1466 3422	1955 2933	2444	$\bar{1}.92$	,95661 ,00535,8		0536 4822	1072 4286	1607 3751	2143 3215	2679
$\bar{1}.68$	,83368 ,00491,7		0492 4425	0983 3934	1475 3442	1967 2950	2459	$\bar{1}.93$	,96197 ,00537,6		0538 4838	1075 4301	1613 3763	2150 3226	2683
$\bar{1}.69$	,83859 ,00492,5		0493 4433	0985 3940	1478 3448	1970 2955	2463	$\bar{1}.94$	,96734 ,00539,6		0540 4826	1079 4317	1619 3777	2158 3238	2698
$\bar{1}.70$	,84351 ,00494,4		0494 4450	0989 3955	1483 3461	1978 2966	2472	$\bar{1}.95$	,97274 ,00541,4		0541 4873	1083 4331	1624 3790	2166 3248	2707
$\bar{1}.71$	,84846 ,00496,3		0496 4467	0993 3970	1489 3474	1985 2978	2482	$\bar{1}.96$	,97815 ,00543,3		0543 4890	1087 4346	1630 3703	2173 3260	2717
$\bar{1}.72$	,85342 ,00498,1		0498 4483	0996 3985	1494 3487	1992 2989	2491	$\bar{1}.97$	,98359 ,00545,2		0545 4907	1090 4362	1636 3716	2181 3271	2726
$\bar{1}.73$	,85840 ,00499,9		0500 4499	1000 3999	1500 3499	2000 2999	2500	$\bar{1}.98$	,98904 ,00547,2		0547 4925	1094 4378	1642 3830	2189 3283	2736
$\bar{1}.74$	,86340 ,00501,9		0502 4517	1004 4005	1506 3513	2008 3011	2510	$\bar{1}.99$	,99451						



General Table II.  $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$ .

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	
$\bar{3}.00$	$\bar{1},77356$ $,00227,0$	,00	0227	0454	0681	0908	$\bar{3}.25$	$\bar{1},83164$ $,00238,5$	,00	0239	0477	0716	0954	1193
$\bar{3}.01$	$\bar{1},77583$ $,00227,4$		0227	0455	0682	0910	$\bar{3}.26$	$\bar{1},83403$ $,00239,0$		0239	0478	0717	0956	1195
$\bar{3}.02$	$\bar{1},77810$ $,00227,8$		0228	0456	0683	0911	$\bar{3}.27$	$\bar{1},83641$ $,00239,5$		0240	0479	0719	0958	1198
$\bar{3}.03$	$\bar{1},78038$ $,00228,3$		0228	0457	0685	0913	$\bar{3}.28$	$\bar{1},83881$ $,00240,0$		0240	0480	0720	0960	1200
$\bar{3}.04$	$\bar{1},78266$ $,00228,6$		0229	0457	0686	0914	$\bar{3}.29$	$\bar{1},84121$ $,00240,5$		0241	0481	0722	0962	1203
$\bar{3}.05$	$\bar{1},78495$ $,00229,2$		0229	0458	0688	0917	$\bar{3}.30$	$\bar{1},84361$ $,00240,9$		0241	0482	0723	0964	1205
$\bar{3}.06$	$\bar{1},78724$ $,00229,6$		0230	0460	0689	0918	$\bar{3}.31$	$\bar{1},84602$ $,00241,5$		0242	0483	0725	0966	1208
$\bar{3}.07$	$\bar{1},78954$ $,00230,0$		0230	0460	0690	0920	$\bar{3}.32$	$\bar{1},84844$ $,00241,9$		0242	0484	0726	0968	1210
$\bar{3}.08$	$\bar{1},79184$ $,00230,5$		0231	0461	0692	0922	$\bar{3}.33$	$\bar{1},85086$ $,00242,5$		0243	0485	0728	0970	1213
$\bar{3}.09$	$\bar{1},79414$ $,00230,8$		0231	0462	0692	0923	$\bar{3}.34$	$\bar{1},85328$ $,00243,2$		0243	0486	0730	0973	1216
$\bar{3}.10$	$\bar{1},79645$ $,00231,4$		0231	0463	0694	0926	$\bar{3}.35$	$\bar{1},85571$ $,00243,6$		0244	0487	0731	0974	1218
$\bar{3}.11$	$\bar{1},79877$ $,00231,8$		0232	0464	0695	0927	$\bar{3}.36$	$\bar{1},85815$ $,00244,1$		0244	0488	0732	0976	1221
$\bar{3}.12$	$\bar{1},80108$ $,00232,2$		0232	0464	0697	0929	$\bar{3}.37$	$\bar{1},86059$ $,00244,7$		0245	0489	0734	0979	1224
$\bar{3}.13$	$\bar{1},80341$ $,00232,7$		0233	0465	0698	0931	$\bar{3}.38$	$\bar{1},86304$ $,00245,2$		0245	0490	0736	0981	1226
$\bar{3}.14$	$\bar{1},80573$ $,00233,2$		0233	0466	0700	0933	$\bar{3}.39$	$\bar{1},86549$ $,00245,8$		0246	0492	0737	0983	1229
$\bar{3}.15$	$\bar{1},80806$ $,00233,6$		0234	0467	0701	0934	$\bar{3}.40$	$\bar{1},86795$ $,00246,2$		0246	0492	0739	0985	1231
$\bar{3}.16$	$\bar{1},81040$ $,00234,2$		0234	0468	0703	0937	$\bar{3}.41$	$\bar{1},87041$ $,00246,9$		0247	0494	0741	0988	1235
$\bar{3}.17$	$\bar{1},81274$ $,00234,5$		0235	0469	0704	0938	$\bar{3}.42$	$\bar{1},87288$ $,00247,4$		0247	0495	0742	0990	1237
$\bar{3}.18$	$\bar{1},81509$ $,00235,0$		0235	0470	0705	0940	$\bar{3}.43$	$\bar{1},87535$ $,00247,9$		0248	0496	0744	0992	1240
$\bar{3}.19$	$\bar{1},81744$ $,00235,5$		0236	0471	0707	0942	$\bar{3}.44$	$\bar{1},87783$ $,00248,5$		0249	0497	0746	0994	1243
$\bar{3}.20$	$\bar{1},81979$ $,00236,0$		0236	0472	0708	0944	$\bar{3}.45$	$\bar{1},88032$ $,00249,1$		0249	0498	0747	0996	1246
$\bar{3}.21$	$\bar{1},82215$ $,00236,5$		0237	0473	0710	0946	$\bar{3}.46$	$\bar{1},88281$ $,00249,7$		0250	0499	0749	0999	1248
$\bar{3}.22$	$\bar{1},82452$ $,00237,0$		0237	0474	0711	0948	$\bar{3}.47$	$\bar{1},88530$ $,00250,2$		0250	0500	0751	1001	1251
$\bar{3}.23$	$\bar{1},82689$ $,00237,4$		0237	0475	0712	0950	$\bar{3}.48$	$\bar{1},88780$ $,00250,8$		0251	0502	0752	1003	1254
$\bar{3}.24$	$\bar{1},82926$ $,00238,0$		0238	0476	0714	0952	$\bar{3}.49$	$\bar{1},89031$ $,00251,4$		0251	0503	0754	1006	1257
			2142	1904	1666	1428				2263	2011	1760	1508	

General Table II.  $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$ .

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.50$	$\bar{1}.89283$ ,00252,0	,00	0252 2268	0504 2016	0756 1764	1008 1512	1260	$\bar{3}.75$	$\bar{1}.95767$ ,00268,0	,00	0268 2412	0536 2144	0804 1876	1072 1608	1340
$\bar{3}.51$	$\bar{1}.89535$ ,00252,6		0253 2273	0505 2021	0758 1768	1010 1516	1263	$\bar{3}.76$	$\bar{1}.96035$ ,00268,5		0269 2417	0537 2148	0806 1880	1074 1611	1343
$\bar{3}.52$	$\bar{1}.89787$ ,00253,1		0253 2278	0506 2025	0759 1772	1012 1519	1266	$\bar{3}.77$	$\bar{1}.96303$ ,00269,3		0269 2424	0539 2154	0808 1885	1077 1616	1347
$\bar{3}.53$	$\bar{1}.90040$ ,00253,7		0254 2283	0507 2030	0761 1776	1015 1522	1269	$\bar{3}.78$	$\bar{1}.96573$ ,00270,0		0270 2430	0540 2160	0810 1890	1080 1620	1350
$\bar{3}.54$	$\bar{1}.90294$ ,00254,3		0254 2289	0509 2034	0763 1780	1017 1526	1272	$\bar{3}.79$	$\bar{1}.96843$ ,00270,7		0271 2436	0541 2166	0812 1895	1083 1624	1354
$\bar{3}.55$	$\bar{1}.90548$ ,00255,0		0255 2295	0510 2040	0765 1785	1020 1530	1275	$\bar{3}.80$	$\bar{1}.97113$ ,00271,3		0271 2442	0543 2170	0814 1899	1085 1628	1357
$\bar{3}.56$	$\bar{1}.90803$ ,00255,5		0256 2300	0511 2044	0767 1789	1022 1533	1278	$\bar{3}.81$	$\bar{1}.97385$ ,00272,1		0272 2449	0544 2177	0816 1905	1088 1633	1361
$\bar{3}.57$	$\bar{1}.91059$ ,00256,2		0256 2306	0512 2050	0769 1793	1025 1537	1281	$\bar{3}.82$	$\bar{1}.97657$ ,00272,9		0273 2456	0546 2183	0819 1910	1092 1637	1365
$\bar{3}.58$	$\bar{1}.91315$ ,00256,8		0257 2311	0514 2054	0770 1798	1027 1541	1284	$\bar{3}.83$	$\bar{1}.97930$ ,00273,4		0273 2461	0547 2187	0820 1914	1094 1640	1367
$\bar{3}.59$	$\bar{1}.91572$ ,00257,4		0257 2317	0515 2059	0772 1802	1030 1544	1287	$\bar{3}.84$	$\bar{1}.98203$ ,00274,3		0274 2469	0549 2194	0823 1920	1097 1646	1372
$\bar{3}.60$	$\bar{1}.91829$ ,00258,0		0258 2322	0516 2064	0774 1806	1032 1548	1290	$\bar{3}.85$	$\bar{1}.98477$ ,00275,0		0275 2475	0550 2200	0825 1925	1100 1650	1375
$\bar{3}.61$	$\bar{1}.92087$ ,00258,7		0259 2328	0517 2070	0776 1811	1035 1552	1294	$\bar{3}.86$	$\bar{1}.98752$ ,00275,8		0276 2482	0552 2206	0827 1931	1103 1655	1379
$\bar{3}.62$	$\bar{1}.92346$ ,00259,2		0259 2333	0518 2074	0778 1814	1037 1555	1296	$\bar{3}.87$	$\bar{1}.99028$ ,00276,5		0277 2489	0553 2212	0830 1936	1106 1659	1383
$\bar{3}.63$	$\bar{1}.92605$ ,00259,9		0260 2339	0520 2079	0780 1819	1040 1559	1300	$\bar{3}.88$	$\bar{1}.99305$ ,00277,3		0277 2496	0555 2218	0832 1941	1109 1664	1387
$\bar{3}.64$	$\bar{1}.92865$ ,00260,6		0261 2345	0521 2085	0782 1824	1042 1564	1303	$\bar{3}.89$	$\bar{1}.99582$ ,00278,0		0278 2502	0556 2224	0834 1946	1112 1668	1390
$\bar{3}.65$	$\bar{1}.93125$ ,00261,2		0261 2351	0522 2090	0784 1828	1045 1567	1306	$\bar{3}.90$	$\bar{1}.99860$ ,00278,8		0279 2509	0558 2230	0836 1952	1115 1673	1394
$\bar{3}.66$	$\bar{1}.93387$ ,00261,8		0262 2356	0524 2094	0785 1833	1047 1571	1309	$\bar{3}.91$	$\bar{1}.00139$ ,00279,5		0280 2516	0559 2236	0839 1957	1118 1677	1398
$\bar{3}.67$	$\bar{1}.93648$ ,00262,5		0263 2363	0525 2100	0788 1838	1050 1575	1313	$\bar{3}.92$	$\bar{1}.00418$ ,00280,3		0280 2523	0561 2242	0841 1962	1121 1682	1402
$\bar{3}.68$	$\bar{1}.93911$ ,00263,2		0263 2369	0526 2106	0790 1842	1053 1579	1316	$\bar{3}.93$	$\bar{1}.00699$ ,00281,1		0281 2530	0562 2249	0843 1968	1124 1687	1406
$\bar{3}.69$	$\bar{1}.94174$ ,00263,8		0264 2374	0528 2110	0791 1847	1055 1583	1319	$\bar{3}.94$	$\bar{1}.00980$ ,00281,9		0282 2537	0564 2255	0846 1973	1128 1691	1410
$\bar{3}.70$	$\bar{1}.94438$ ,00264,4		0264 2380	0529 2115	0793 1851	1058 1586	1322	$\bar{3}.95$	$\bar{1}.01262$ ,00282,7		0283 2544	0565 2262	0848 1979	1131 1696	1414
$\bar{3}.71$	$\bar{1}.94702$ ,00265,1		0265 2386	0530 2121	0795 1856	1060 1591	1326	$\bar{3}.96$	$\bar{1}.01544$ ,00283,4		0283 2551	0567 2267	0850 1984	1134 1700	1417
$\bar{3}.72$	$\bar{1}.94967$ ,00265,8		0266 2392	0532 2126	0797 1861	1063 1595	1329	$\bar{3}.97$	$\bar{1}.01828$ ,00284,2		0284 2558	0568 2274	0853 1989	1137 1705	1421
$\bar{3}.73$	$\bar{1}.95233$ ,00266,5		0267 2399	0533 2132	0800 1866	1066 1599	1333	$\bar{3}.98$	$\bar{1}.02112$ ,00285,0		0285 2565	0570 2280	0855 1995	1140 1710	1425
$\bar{3}.74$	$\bar{1}.95500$ ,00267,2		0267 2405	0534 2138	0802 1870	1069 1603	1336	$\bar{3}.99$	$\bar{1}.02397$ ,00285,9		0286 2573	0572 2287	0858 2001	1144 1715	1430

General Table II.  $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^7-1}\right)$ .

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{z}.00$	,02683	,00	0287	0573	0860	1147	1434	$\bar{z}.25$	,10107	,00	0309	0618	0926	1235	1544
	,00286,7		2580	2294	2007	1720			,00308,8		2779	2470	2162	1853	
$\bar{z}.01$	,02969		0288	0575	0863	1150	1438	$\bar{z}.26$	,10416		0310	0620	0929	1239	1549
	,00287,5		2588	2300	2013	1725			,00309,8		2788	2478	2169	1859	
$\bar{z}.02$	,03257		0288	0577	0865	1153	1442	$\bar{z}.27$	,10726		0311	0621	0932	1243	1554
	,00288,3		2595	2306	2018	1730			,00310,7		2796	2486	2175	1864	
$\bar{z}.03$	,03545		0289	0578	0867	1156	1446	$\bar{z}.28$	,11037		0312	0623	0935	1247	1559
	,00289,1		2602	2313	2024	1735			,00311,7		2805	2494	2182	1870	
$\bar{z}.04$	,03834		0290	0580	0870	1160	1450	$\bar{z}.29$	,11348		0313	0625	0938	1251	1564
	,00290,0		2610	2320	2030	1740			,00312,7		2814	2502	2189	1876	
$\bar{z}.05$	,04124		0291	0582	0872	1163	1454	$\bar{z}.30$	,11661		0314	0627	0941	1255	1569
	,00290,8		2617	2326	2036	1745			,00313,7		2823	2510	2196	1882	
$\bar{z}.06$	,04415		0292	0583	0875	1167	1459	$\bar{z}.31$	,11975		0315	0629	0944	1258	1573
	,00291,7		2625	2334	2043	1750			,00314,6		2831	2517	2202	1888	
$\bar{z}.07$	,04707		0293	0585	0878	1170	1463	$\bar{z}.32$	,12289		0316	0631	0947	1262	1578
	,00292,5		2633	2340	2048	1755			,00315,6		2840	2525	2209	1894	
$\bar{z}.08$	,04999		0293	0587	0880	1174	1467	$\bar{z}.33$	,12605		0317	0633	0950	1267	1583
	,00293,4		2641	2347	2054	1760			,00316,7		2850	2534	2216	1900	
$\bar{z}.09$	,05293		0294	0588	0883	1177	1471	$\bar{z}.34$	,12921		0318	0635	0953	1271	1589
	,00294,2		2648	2354	2059	1765			,00317,7		2859	2542	2224	1906	
$\bar{z}.10$	,05587		0295	0590	0885	1180	1476	$\bar{z}.35$	,13239		0319	0637	0956	1275	1594
	,00295,1		2656	2361	2066	1771			,00318,7		2868	2550	2231	1912	
$\bar{z}.11$	,05882		0296	0592	0888	1184	1480	$\bar{z}.36$	,13558		0320	0639	0959	1279	1599
	,00296,0		2664	2368	2072	1776			,00319,7		2877	2558	2238	1918	
$\bar{z}.12$	,06178		0297	0594	0890	1187	1484	$\bar{z}.37$	,13898		0321	0641	0962	1283	1604
	,00296,8		2671	2374	2078	1781			,00320,7		2886	2566	2245	1924	
$\bar{z}.13$	,06475		0298	0596	0893	1191	1489	$\bar{z}.38$	,14198		0322	0644	0965	1287	1609
	,00297,8		2680	2382	2085	1787			,00321,8		2896	2574	2253	1931	
$\bar{z}.14$	,06772		0299	0597	0896	1194	1493	$\bar{z}.39$	,14520		0323	0646	0968	1291	1614
	,00298,6		2687	2389	2090	1792			,00322,8		2905	2582	2260	1937	
$\bar{z}.15$	,07071		0300	0599	0899	1198	1498	$\bar{z}.40$	,14843		0324	0648	0972	1296	1620
	,00299,5		2696	2396	2097	1797			,00323,9		2915	2591	2267	1943	
$\bar{z}.16$	,07370		0300	0601	0901	1202	1502	$\bar{z}.41$	,15167		0325	0650	0975	1300	1625
	,00300,4		2704	2403	2103	1802			,00324,9		2924	2599	2274	1949	
$\bar{z}.17$	,07671		0301	0603	0904	1205	1507	$\bar{z}.42$	,15492		0326	0652	0978	1304	1630
	,00301,3		2712	2410	2109	1808			,00325,9		2933	2607	2281	1955	
$\bar{z}.18$	,07972		0302	0604	0907	1209	1511	$\bar{z}.43$	,15818		0327	0654	0981	1308	1636
	,00302,2		2720	2418	2115	1813			,00327,1		2944	2617	2290	1963	
$\bar{z}.19$	,08274		0303	0606	0902	1212	1516	$\bar{z}.44$	,16145		0328	0656	0984	1312	1641
	,00303,1		2728	2425	2122	1819			,00328,1		2953	2625	2297	1969	
$\bar{z}.20$	,08577		0304	0608	0912	1216	1521	$\bar{z}.45$	,16473		0329	0658	0988	1317	1646
	,00304,1		2737	2433	2129	1825			,00329,2		2963	2634	2304	1975	
$\bar{z}.21$	,08882		0305	0610	0915	1220	1525	$\bar{z}.46$	,16802		0330	0661	0991	1321	1652
	,00305,0		2745	2440	2135	1830			,00330,3		2973	2642	2312	1982	
$\bar{z}.22$	,09187		0306	0612	0918	1224	1530	$\bar{z}.47$	,17132		0331	0663	0994	1326	1657
	,00305,9		2753	2447	2141	1835			,00331,4		2983	2651	2320	1988	
$\bar{z}.23$	,09493		0307	0614	0921	1228	1535	$\bar{z}.48$	,17464		0333	0665	0998	1330	1663
	,00306,9		2762	2455	2148	1841			,00332,5		2993	2660	2328	1995	
$\bar{z}.24$	,09799		0308	0616	0923	1231	1539	$\bar{z}.49$	,17796		0334	0667	1001	1334	1668
	,00307,8		2770	2462	2155	1847			,00333,6		3002	2669	2335	2002	

General Table II.  $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$ .

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5
$\bar{z}.50$	,18130 ,00334,7	,00 0335 3012	0669 2678	1004 2343	1339 2008	1674	$\bar{z}.75$	,26850 ,00364,7	,00 0365 3282	0729 2918	1094 2553	1459 2188	1824
$\bar{z}.51$	,18464 ,00335,8	0336 3022	0672 2686	1007 2351	1343 2015	1679	$\bar{z}.76$	,27214 ,00366,2	0366 3296	0732 2930	1099 2563	1465 2197	1831
$\bar{z}.52$	,18880 ,00336,9	0337 3032	0674 2695	1011 2358	1348 2021	1685	$\bar{z}.77$	,27581 ,00367,4	0367 3307	0735 2939	1102 2572	1470 2204	1837
$\bar{z}.53$	,19137 ,00338,1	0338 3043	0676 2705	1014 2367	1352 2029	1691	$\bar{z}.78$	,27948 ,00368,7	0369 3318	0737 2950	1106 2581	1475 2212	1844
$\bar{z}.54$	,19475 ,00339,2	0339 3053	0678 2714	1018 2374	1357 2035	1696	$\bar{z}.79$	,28317 ,00370,1	0370 3331	0740 2961	1110 2591	1480 2221	1851
$\bar{z}.55$	,19815 ,00340,4	0340 3064	0681 2723	1021 2383	1362 2042	1702	$\bar{z}.80$	,28687 ,00371,3	0370 3343	0743 2970	1114 2599	1485 2228	1857
$\bar{z}.56$	,20155 ,00341,5	0342 3074	0683 2732	1025 2391	1366 2049	1708	$\bar{z}.81$	,29058 ,00372,8	0373 3355	0746 2982	1118 2610	1491 2237	1864
$\bar{z}.57$	,20496 ,00342,7	0343 3084	0685 2742	1028 2399	1371 2056	1714	$\bar{z}.82$	,29431 ,00373,9	0374 3365	0748 2991	1122 2617	1496 2243	1870
$\bar{z}.58$	,20839 ,00343,9	0344 3095	0688 2751	1032 2407	1376 2063	1720	$\bar{z}.83$	,29805 ,00375,4	0375 3379	0751 3003	1126 2628	1502 2252	1877
$\bar{z}.59$	,21183 ,00345,0	0345 3105	0690 2760	1035 2415	1380 2070	1725	$\bar{z}.84$	,30180 ,00376,7	0377 3390	0753 3014	1130 2637	1507 2260	1884
$\bar{z}.60$	,21528 ,00346,2	0346 3116	0692 2770	1039 2423	1385 2077	1731	$\bar{z}.85$	,30557 ,00378,1	0378 3403	0756 3025	1134 2647	1512 2269	1891
$\bar{z}.61$	,21874 ,00347,4	0347 3127	0695 2779	1042 2432	1390 2084	1737	$\bar{z}.86$	,30935 ,00379,4	0379 3415	0759 3035	1138 2656	1518 2276	1897
$\bar{z}.62$	,22222 ,00348,7	0349 3138	0697 2790	1046 2441	1395 2092	1744	$\bar{z}.87$	,31314 ,00380,8	0381 3427	0762 3046	1142 2666	1523 2285	1904
$\bar{z}.63$	,22570 ,00349,7	0350 3147	0699 2798	1049 2448	1399 2098	1749	$\bar{z}.88$	,31695 ,00382,2	0382 3440	0764 3058	1146 2675	1529 2293	1911
$\bar{z}.64$	,22920 ,00351,1	0351 3160	0702 2809	1053 2458	1404 2107	1756	$\bar{z}.89$	,32077 ,00383,6	0384 3452	0767 3069	1151 2685	1534 2302	1918
$\bar{z}.65$	,23271 ,00352,3	0352 3171	0705 2818	1057 2466	1409 2113	1762	$\bar{z}.90$	,32461 ,00385,0	0385 3465	0770 3080	1155 2695	1540 2310	1925
$\bar{z}.66$	,23623 ,00353,5	0354 3182	0707 2828	1061 2475	1414 2121	1768	$\bar{z}.91$	,32846 ,00386,3	0386 3477	0773 3090	1159 2704	1545 2318	1932
$\bar{z}.67$	,23977 ,00354,7	0355 3192	0709 2838	1064 2483	1419 2128	1774	$\bar{z}.92$	,33232 ,00387,8	0388 3490	0776 3102	1163 2715	1551 2327	1939
$\bar{z}.68$	,24332 ,00356,0	0356 3204	0712 2848	1068 2492	1424 2136	1780	$\bar{z}.93$	,33620 ,00389,2	0389 3503	0778 3114	1168 2724	1557 2335	1946
$\bar{z}.69$	,24688 ,00357,2	0357 3215	0714 2858	1072 2500	1429 2143	1786	$\bar{z}.94$	,34009 ,00390,6	0391 3515	0781 3125	1172 2734	1562 2344	1953
$\bar{z}.70$	,25045 ,00358,4	0358 3226	0717 2867	1075 2509	1434 2150	1792	$\bar{z}.95$	,34400 ,00392,0	0392 3528	0784 3136	1176 2744	1568 2352	1960
$\bar{z}.71$	,25403 ,00359,7	0360 3237	0719 2878	1079 2518	1439 2158	1799	$\bar{z}.96$	,34792 ,00393,5	0394 3542	0787 3148	1181 2755	1574 2361	1968
$\bar{z}.72$	,25763 ,00361,0	0361 3249	0722 2888	1083 2527	1444 2166	1805	$\bar{z}.97$	,35185 ,00394,9	0395 3554	0790 3159	1185 2764	1580 2369	1975
$\bar{z}.73$	,26124 ,00362,2	0362 3260	0724 2898	1087 2535	1449 2173	1811	$\bar{z}.98$	,35580 ,00396,3	0396 3567	0793 3170	1189 2774	1585 2378	1982
$\bar{z}.74$	,26486 ,00363,5	0364 3272	0727 2908	1091 2545	1454 2181	1817	$\bar{z}.99$	,35976 ,00397,8	0398 3580	0796 3182	1193 2785	1591 2387	1989

General Table II.  $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$ .

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5
$\bar{1}.00$	,36374 ,00399,3	,00 0399	0799	1198	1597	1997	$\bar{1}.25$	,48811 ,00438,0	,00 0438	0876	1314	1752	2190
$\bar{1}.01$	,36773 ,00400,7	0401 3606	0801	1202	1603	2004	$\bar{1}.26$	,47249 ,00439,4	0439 3955	0879	1318	1758	2197
$\bar{1}.02$	,37174 ,00402,2	0402 3620	0804	1207	1609	2011	$\bar{1}.27$	,47689 ,00441,2	0441 3971	0882	1324	1765	2206
$\bar{1}.03$	,37576 ,00403,7	0404 3633	0807	1211	1615	2019	$\bar{1}.28$	,48130 ,00442,9	0443 3986	0886	1329	1772	2215
$\bar{1}.04$	,37980 ,00405,4	0405 3649	0811	1216	1622	2027	$\bar{1}.29$	,48573 ,00444,5	0445 4001	0889	1334	1778	2223
$\bar{1}.05$	,38385 ,00406,4	0406 3658	0813	1219	1626	2032	$\bar{1}.30$	,49017 ,00446,2	0446 4016	0892	1339	1785	2231
$\bar{1}.06$	,38792 ,00408,2	0408 3674	0816	1225	1633	2041	$\bar{1}.31$	,49463 ,00447,8	0448 4030	0896	1343	1791	2239
$\bar{1}.07$	,39200 ,00409,7	0410 3687	0819	1229	1639	2050	$\bar{1}.32$	,49911 ,00449,5	0450 4046	0899	1349	1798	2248
$\bar{1}.08$	,39610 ,00411,2	0411 3701	0822	1234	1645	2056	$\bar{1}.33$	,50361 ,00451,1	0451 4060	0902	1353	1804	2256
$\bar{1}.09$	,40021 ,00412,8	0413 3715	0826	1238	1651	2064	$\bar{1}.34$	,50812 ,00452,9	0453 4076	0909	1359	1812	2265
$\bar{1}.10$	,40434 ,00414,2	0414 3728	0828	1243	1657	2071	$\bar{1}.35$	,51265 ,00454,5	0455 4090	0909	1364	1818	2273
$\bar{1}.11$	,40848 ,00415,8	0416 3742	0832	1247	1663	2079	$\bar{1}.36$	,51719 ,00456,2	0456 4106	0912	1369	1825	2281
$\bar{1}.12$	,41264 ,00417,3	0417 3756	0835	1252	1669	2087	$\bar{1}.37$	,52175 ,00457,8	0458 4120	0916	1373	1831	2289
$\bar{1}.13$	,41681 ,00419,1	0419 3772	0838	1257	1676	2096	$\bar{1}.38$	,52633 ,00459,7	0460 4137	0919	1379	1839	2299
$\bar{1}.14$	,42100 ,00420,2	0420 3782	0840	1261	1681	2101	$\bar{1}.39$	,53093 ,00461,4	0461 4153	0923	1384	1846	2307
$\bar{1}.15$	,42520 ,00421,9	0422 3797	0844	1266	1688	2110	$\bar{1}.40$	,53554 ,00462,9	0463 4166	0926	1389	1852	2315
$\bar{1}.16$	,42942 ,00423,6	0424 3812	0847	1271	1694	2118	$\bar{1}.41$	,54017 ,00464,8	0465 4183	0930	1394	1859	2324
$\bar{1}.17$	,43366 ,00425,1	0425 3826	0850	1275	1700	2126	$\bar{1}.42$	,54482 ,00466,5	0467 4199	0933	1400	1866	2333
$\bar{1}.18$	,43791 ,00426,8	0427 3841	0854	1280	1707	2134	$\bar{1}.43$	,54948 ,00468,1	0468 4213	0936	1404	1872	2341
$\bar{1}.19$	,44218 ,00428,1	0428 3853	0856	1284	1712	2141	$\bar{1}.44$	,55417 ,00469,9	0470 4229	0940	1410	1880	2350
$\bar{1}.20$	,44646 ,00429,9	0430 3869	0860	1290	1720	2150	$\bar{1}.45$	,55886 ,00471,7	0472 4245	0943	1415	1887	2359
$\bar{1}.21$	,45076 ,00431,5	0432 3884	0863	1295	1726	2158	$\bar{1}.46$	,56358 ,00473,6	0474 4262	0947	1421	1894	2368
$\bar{1}.22$	,45507 ,00433,1	0433 3898	0866	1299	1732	2166	$\bar{1}.47$	,56832 ,00475,1	0475 4276	0950	1425	1900	2376
$\bar{1}.23$	,45940 ,00434,7	0435 3912	0869	1304	1739	2174	$\bar{1}.48$	,57307 ,00476,9	0477 4292	0954	1431	1901	2385
$\bar{1}.24$	,46375 ,00436,3	0436 3927	0873	1309	1745	2182	$\bar{1}.49$	,57784 ,00478,7	0479 4308	0957	1436	1915	2394



General Table III.  $\lambda(a^s), \lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$ .

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.00$	$\bar{1},52519$ $,00266,3$	.00	0266	0533	0799	1065	1332	$\bar{3}.25$	$\bar{1},59300$ $,00276,9$	.00	0277	0554	0831	1108	1385
$\bar{3}.01$	$\bar{1},52786$ $,00266,5$		0267	0533	0800	1066	1333	$\bar{3}.26$	$\bar{1},59577$ $,00277,4$		0277	0555	0832	1110	1387
$\bar{3}.02$	$\bar{1},53052$ $,00267,2$		0267	0534	0802	1069	1336	$\bar{3}.27$	$\bar{1},59855$ $,00277,9$		0278	0556	0834	1112	1390
$\bar{3}.03$	$\bar{1},53319$ $,00267,5$		0268	0535	0803	1070	1338	$\bar{3}.28$	$\bar{1},60133$ $,00278,4$		0278	0557	0835	1114	1392
$\bar{3}.04$	$\bar{1},53587$ $,00267,9$		0268	0536	0804	1072	1340	$\bar{3}.29$	$\bar{1},60411$ $,00278,8$		0279	0558	0836	1115	1394
$\bar{3}.05$	$\bar{1},53855$ $,00268,3$		0268	0537	0805	1073	1342	$\bar{3}.30$	$\bar{1},60690$ $,00279,3$		0279	0559	0838	1117	1397
$\bar{3}.06$	$\bar{1},54123$ $,00268,7$		0269	0537	0806	1075	1344	$\bar{3}.31$	$\bar{1},60969$ $,00279,8$		0280	0560	0839	1119	1399
$\bar{3}.07$	$\bar{1},54392$ $,00269,1$		0269	0538	0807	1076	1346	$\bar{3}.32$	$\bar{1},61249$ $,00280,2$		0280	0560	0841	1121	1401
$\bar{3}.08$	$\bar{1},54661$ $,00269,5$		0270	0539	0809	1078	1348	$\bar{3}.33$	$\bar{1},61529$ $,00280,8$		0281	0562	0842	1123	1404
$\bar{3}.09$	$\bar{1},54930$ $,00269,9$		0270	0540	0810	1080	1350	$\bar{3}.34$	$\bar{1},61810$ $,00281,3$		0281	0563	0844	1125	1407
$\bar{3}.10$	$\bar{1},55200$ $,00270,3$		0270	0541	0811	1081	1352	$\bar{3}.35$	$\bar{1},62091$ $,00281,8$		0282	0564	0845	1127	1409
$\bar{3}.11$	$\bar{1},55470$ $,00270,7$		0271	0541	0812	1083	1354	$\bar{3}.36$	$\bar{1},62373$ $,00282,2$		0282	0564	0847	1129	1411
$\bar{3}.12$	$\bar{1},55741$ $,00271,2$		0271	0542	0814	1085	1356	$\bar{3}.37$	$\bar{1},62655$ $,00282,8$		0283	0566	0848	1131	1414
$\bar{3}.13$	$\bar{1},56012$ $,00271,6$		0272	0543	0815	1086	1358	$\bar{3}.38$	$\bar{1},62938$ $,00283,3$		0283	0567	0850	1133	1417
$\bar{3}.14$	$\bar{1},56284$ $,00272,0$		0272	0544	0816	1088	1360	$\bar{3}.39$	$\bar{1},63221$ $,00283,8$		0284	0568	0851	1135	1419
$\bar{3}.15$	$\bar{1},56556$ $,00272,5$		0273	0545	0818	1090	1363	$\bar{3}.40$	$\bar{1},63505$ $,00284,3$		0284	0569	0853	1137	1422
$\bar{3}.16$	$\bar{1},56828$ $,00272,9$		0273	0546	0819	1092	1365	$\bar{3}.41$	$\bar{1},63789$ $,00284,8$		0285	0570	0854	1139	1424
$\bar{3}.17$	$\bar{1},57101$ $,00273,3$		0273	0547	0820	1093	1367	$\bar{3}.42$	$\bar{1},64074$ $,00285,3$		0285	0571	0856	1141	1427
$\bar{3}.18$	$\bar{1},57375$ $,00273,8$		0274	0548	0821	1095	1369	$\bar{3}.43$	$\bar{1},64359$ $,00285,8$		0286	0572	0857	1143	1429
$\bar{3}.19$	$\bar{1},57648$ $,00274,2$		0274	0548	0823	1097	1371	$\bar{3}.44$	$\bar{1},64645$ $,00286,3$		0286	0573	0859	1145	1432
$\bar{3}.20$	$\bar{1},57923$ $,00274,7$		0275	0549	0824	1099	1374	$\bar{3}.45$	$\bar{1},64931$ $,00286,9$		0287	0574	0861	1148	1435
$\bar{3}.21$	$\bar{1},58197$ $,00275,1$		0275	0550	0825	1100	1376	$\bar{3}.46$	$\bar{1},65218$ $,00287,4$		0287	0575	0862	1150	1437
$\bar{3}.22$	$\bar{1},58472$ $,00275,6$		0276	0551	0827	1102	1378	$\bar{3}.47$	$\bar{1},65506$ $,00288,0$		0288	0576	0864	1152	1440
$\bar{3}.23$	$\bar{1},58748$ $,00276,0$		0276	0552	0828	1104	1380	$\bar{3}.48$	$\bar{1},65794$ $,00288,5$		0289	0577	0866	1154	1443
$\bar{3}.24$	$\bar{1},59024$ $,00276,5$		0277	0553	0830	1106	1383	$\bar{3}.49$	$\bar{1},66082$ $,00289,0$		0289	0578	0867	1156	1445
			2489	2212	1936	1659					2601	2312	2023	1734	

General Table III.  $\lambda(a^s), \lambda\left(\frac{1-a^s}{a^s-1}\right)$ .

$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^s-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^s)$	$\lambda\left(\frac{1-a^s}{a^s-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.50$	$\bar{1}.66371$ ,00289,6	,00	0290 2606	0579 2317	0869 2027	1158 1738	1448	$\bar{3}.75$	$\bar{1}.73787$ ,00304,7	,00	0305 2742	0609 2438	0914 2133	1219 1828	1524
$\bar{3}.51$	$\bar{1}.66661$ ,00290,1		0290 2611	0580 2321	0870 2031	1160 1741	1451	$\bar{3}.76$	$\bar{1}.74091$ ,00305,3		0305 2748	0611 2442	0916 2137	1221 1832	1527
$\bar{3}.52$	$\bar{1}.66951$ ,00290,7		0291 2616	0581 2326	0872 2035	1163 1744	1454	$\bar{3}.77$	$\bar{1}.74393$ ,00306,0		0306 2754	0612 2448	0918 2142	1224 1836	1530
$\bar{3}.53$	$\bar{1}.67242$ ,00291,4		0291 2623	0583 2331	0874 2040	1166 1748	1457	$\bar{3}.78$	$\bar{1}.74702$ ,00306,6		0307 2759	0613 2453	0920 2146	1226 1840	1533
$\bar{3}.54$	$\bar{1}.67533$ ,00291,8		0292 2626	0584 2334	0875 2043	1167 1751	1459	$\bar{3}.79$	$\bar{1}.75009$ ,00307,4		0307 2767	0615 2459	0922 2152	1230 1844	1537
$\bar{3}.55$	$\bar{1}.67825$ ,00292,4		0292 2632	0585 2339	0877 2047	1170 1754	1462	$\bar{3}.80$	$\bar{1}.75316$ ,00308,0		0308 2772	0616 2464	0924 2156	1232 1848	1540
$\bar{3}.56$	$\bar{1}.68117$ ,00292,9		0293 2636	0586 2343	0879 2050	1172 1757	1465	$\bar{3}.81$	$\bar{1}.75624$ ,00308,8		0309 2779	0618 2470	0926 2162	1235 1853	1544
$\bar{3}.57$	$\bar{1}.68410$ ,00293,7		0294 2643	0587 2350	0881 2056	1175 1762	1469	$\bar{3}.82$	$\bar{1}.75933$ ,00309,3		0309 2784	0609 2474	0928 2165	1237 1856	1547
$\bar{3}.58$	$\bar{1}.68704$ ,00294,1		0294 2647	0588 2333	0882 2059	1176 1765	1471	$\bar{3}.83$	$\bar{1}.76243$ ,00310,1		0310 2771	0620 2481	0930 2171	1240 1861	1551
$\bar{3}.59$	$\bar{1}.68998$ ,00294,7		0295 2652	0589 2358	0884 2063	1179 1768	1474	$\bar{3}.84$	$\bar{1}.76553$ ,00310,8		0311 2797	0622 2486	0932 2176	1243 1865	1554
$\bar{3}.60$	$\bar{1}.69293$ ,00295,3		0295 2658	0591 2362	0886 2067	1181 1772	1477	$\bar{3}.85$	$\bar{1}.76863$ ,00311,4		0311 2803	0623 2491	0934 2180	1246 1868	1557
$\bar{3}.61$	$\bar{1}.69588$ ,00295,9		0296 2663	0592 2367	0888 2071	1184 1775	1480	$\bar{3}.86$	$\bar{1}.77175$ ,00312,2		0312 2810	0624 2498	0937 2185	1249 1873	1561
$\bar{3}.62$	$\bar{1}.69884$ ,00296,5		0297 2669	0593 2372	0890 2076	1186 1779	1483	$\bar{3}.87$	$\bar{1}.77487$ ,00313,0		0313 2817	0626 2504	0939 2191	1252 1878	1565
$\bar{3}.63$	$\bar{1}.70180$ ,00297,1		0297 2674	0594 2377	0891 2080	1188 1783	1486	$\bar{3}.88$	$\bar{1}.77800$ ,00313,6		0314 2822	0627 2509	0941 2195	1254 1882	1568
$\bar{3}.64$	$\bar{1}.70477$ ,00297,7		0298 2679	0595 2382	0893 2084	1191 1786	1489	$\bar{3}.89$	$\bar{1}.78113$ ,00314,3		0314 2829	0629 2514	0943 2200	1257 1886	1572
$\bar{3}.65$	$\bar{1}.70775$ ,00298,4		0298 2686	0597 2387	0895 2089	1194 1790	1492	$\bar{3}.90$	$\bar{1}.78428$ ,00315,0		0315 2835	0630 2520	0945 2205	1260 1890	1575
$\bar{3}.66$	$\bar{1}.71074$ ,00299,88		0299 2689	0598 2390	0896 2092	1195 1793	1494	$\bar{3}.91$	$\bar{1}.78743$ ,00315,8		0316 2842	0632 2526	0947 2211	1263 1895	1579
$\bar{3}.67$	$\bar{1}.71372$ ,00299,6		0300 2696	0599 2397	0899 2097	1198 1798	1498	$\bar{3}.92$	$\bar{1}.79059$ ,00316,4		0316 2848	0633 2531	0949 2215	1266 1898	1582
$\bar{3}.68$	$\bar{1}.71672$ ,00300,2		0300 2702	0600 2402	0901 2101	1201 1801	1501	$\bar{3}.93$	$\bar{1}.79375$ ,00317,3		0317 2856	0635 2538	0952 2221	1269 1904	1587
$\bar{3}.69$	$\bar{1}.71972$ ,00300,8		0301 2707	0602 2406	0902 2106	1203 1805	1504	$\bar{3}.94$	$\bar{1}.79692$ ,00318,0		0318 2862	0636 2544	0954 2226	1272 1908	1590
$\bar{3}.70$	$\bar{1}.72273$ ,00301,4		0301 2713	0603 2411	0904 2110	1206 1808	1507	$\bar{3}.95$	$\bar{1}.80010$ ,00318,7		0319 2868	0637 2550	0956 2231	1275 1912	1594
$\bar{3}.71$	$\bar{1}.72574$ ,00302,0		0302 2718	0604 2416	0906 2114	1208 1812	1510	$\bar{3}.96$	$\bar{1}.80329$ ,00319,5		0320 2876	0639 2556	0959 2237	1278 1917	1598
$\bar{3}.72$	$\bar{1}.72876$ ,00302,7		0303 2724	0605 2422	0908 2119	1211 1816	1514	$\bar{3}.97$	$\bar{1}.80648$ ,00320,3		0320 2883	0641 2562	0961 2242	1281 1922	1602
$\bar{3}.73$	$\bar{1}.73179$ ,00303,4		0303 2731	0607 2427	0910 2124	1214 1820	1517	$\bar{3}.98$	$\bar{1}.80969$ ,00321,0		0321 2889	0642 2568	0963 2247	1284 1926	1605
$\bar{3}.74$	$\bar{1}.73483$ ,00304,0		0304 2736	0608 2432	0912 2128	1216 1824	1520	$\bar{3}.99$	$\bar{1}.81290$ ,00321,8		0322 2896	0644 2574	0965 2253	1287 1931	1609



General Table III.  $\lambda (a^5), \lambda \left( \frac{1-a^5}{a^5-1} \right)$ .

$\lambda(a^5)$	$\lambda \left( \frac{1-a^5}{a^5-1} \right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^5)$	$\lambda \left( \frac{1-a^5}{a^5-1} \right)$	1 9	2 8	3 7	4 6	5		
$\bar{z}.00$	$\bar{1},81612$ ,00322,5	,00	0323	0645	0968	1290	1613	$\bar{z}.25$	$\bar{1},89923$ ,00343,8	,00	0344	0688	1031	1375	1719
$\bar{z}.01$	$\bar{1},81934$ ,00323,4		0323	0647	0970	1294	1617	$\bar{z}.26$	$\bar{1},90267$ ,00344,7		0345	0689	1034	1379	1724
$\bar{z}.02$	$\bar{1},82257$ ,00324,2		0324	0648	0973	1297	1621	$\bar{z}.27$	$\bar{1},90612$ ,00345,7		0346	0691	1037	1383	1729
$\bar{z}.03$	$\bar{1},82582$ ,00324,9		0325	0650	0975	1300	1625	$\bar{z}.28$	$\bar{1},90958$ ,00346,6		0347	0693	1040	1386	1733
$\bar{z}.04$	$\bar{1},82907$ ,00325,8		0326	0652	0977	1303	1629	$\bar{z}.29$	$\bar{1},91304$ ,00347,6		0348	0695	1043	1390	1738
$\bar{z}.05$	$\bar{1},83232$ ,00326,5		0327	0653	0980	1306	1633	$\bar{z}.30$	$\bar{1},91652$ ,00348,5		0349	0697	1046	1394	1743
$\bar{z}.06$	$\bar{1},83559$ ,00327,3		0327	0655	0982	1309	1637	$\bar{z}.31$	$\bar{1},92000$ ,00349,3		0349	0699	1048	1397	1747
$\bar{z}.07$	$\bar{1},83886$ ,00328,2		0328	0656	0985	1313	1641	$\bar{z}.32$	$\bar{1},92349$ ,00350,7		0351	0701	1052	1403	1754
$\bar{z}.08$	$\bar{1},84214$ ,00329,0		0329	0658	0987	1316	1645	$\bar{z}.33$	$\bar{1},92700$ ,00351,4		0351	0703	1054	1406	1757
$\bar{z}.09$	$\bar{1},84543$ ,00329,8		0330	0660	0989	1319	1649	$\bar{z}.34$	$\bar{1},93052$ ,00352,4		0352	0705	1057	1410	1762
$\bar{z}.10$	$\bar{1},84873$ ,00330,6		0331	0661	0992	1322	1653	$\bar{z}.35$	$\bar{1},93404$ ,00353,3		0353	0707	1060	1413	1767
$\bar{z}.11$	$\bar{1},85204$ ,00331,5		0332	0663	0995	1326	1658	$\bar{z}.36$	$\bar{1},93757$ ,00354,3		0354	0709	1063	1417	1771
$\bar{z}.12$	$\bar{1},85553$ ,00332,4		0332	0665	0997	1330	1662	$\bar{z}.37$	$\bar{1},94112$ ,00355,4		0355	0711	1066	1422	1777
$\bar{z}.13$	$\bar{1},85868$ ,00333,2		0333	0666	1000	1333	1666	$\bar{z}.38$	$\bar{1},94467$ ,00356,4		0356	0713	1069	1426	1782
$\bar{z}.14$	$\bar{1},86201$ ,00334,0		0334	0668	1002	1336	1670	$\bar{z}.39$	$\bar{1},94823$ ,00357,3		0357	0715	1072	1429	1787
$\bar{z}.15$	$\bar{1},86535$ ,00334,9		0335	0670	1005	1340	1675	$\bar{z}.40$	$\bar{1},95181$ ,00358,4		0358	0717	1075	1434	1792
$\bar{z}.16$	$\bar{1},86870$ ,00335,7		0336	0671	1007	1343	1679	$\bar{z}.41$	$\bar{1},95539$ ,00359,4		0359	0719	1078	1438	1797
$\bar{z}.17$	$\bar{1},87205$ ,00336,6		0337	0673	1010	1346	1683	$\bar{z}.42$	$\bar{1},95898$ ,00360,4		0360	0721	1081	1442	1802
$\bar{z}.18$	$\bar{1},87542$ ,00337,5		0338	0675	1013	1350	1688	$\bar{z}.43$	$\bar{1},96259$ ,00361,4		0361	0723	1084	1446	1807
$\bar{z}.19$	$\bar{1},87880$ ,00338,4		0338	0677	1015	1354	1692	$\bar{z}.44$	$\bar{1},96620$ ,00362,4		0362	0725	1087	1450	1812
$\bar{z}.20$	$\bar{1},88218$ ,00339,3		0339	0679	1018	1357	1697	$\bar{z}.45$	$\bar{1},96983$ ,00363,4		0363	0727	1090	1454	1817
$\bar{z}.21$	$\bar{1},88557$ ,00340,2		0340	0680	1021	1361	1701	$\bar{z}.46$	$\bar{1},97346$ ,00364,6		0365	0729	1094	1458	1823
$\bar{z}.22$	$\bar{1},88897$ ,00341,0		0341	0682	1023	1364	1705	$\bar{z}.47$	$\bar{1},97711$ ,00365,6		0366	0731	1097	1462	1828
$\bar{z}.23$	$\bar{1},89238$ ,00342,1		0342	0684	1026	1368	1711	$\bar{z}.48$	$\bar{1},98076$ ,00366,7		0367	0733	1100	1467	1834
$\bar{z}.24$	$\bar{1},89581$ ,00342,8		0343	0686	1028	1371	1714	$\bar{z}.49$	$\bar{1},98443$ ,00367,8		0368	0735	1103	1471	1839
			3085	2742	2400	2057					3310	2942	2575	2207	

General Table III.  $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$ .

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{z}.50$	$\bar{1},98811$	,00	0369	0738	1106	1475	1844	$\bar{z}.75$	,08373	,00	0398	0796	1194	1592	1990
	,00368,8		3319	2950	2582	2213			,00397,9		3581	3183	2785	2387	
$\bar{z}.51$	$\bar{1},99179$		0370	0740	1109	1480	1850	$\bar{z}.76$	,08771		0399	0799	1198	1598	1997
	,00369,9		3329	2959	2589	2219			,00399,4		3595	3195	2796	2396	
$\bar{z}.52$	$\bar{1},99549$		0371	0742	1113	1484	1855	$\bar{z}.77$	,09170		0401	0801	1202	1602	2003
	,00371,0		3339	2968	2597	2226			,00400,6		3605	3205	2804	2404	
$\bar{z}.53$	$\bar{1},99920$		0372	0744	1116	1488	1861	$\bar{z}.78$	,09571		0402	0803	1205	1607	2009
	,00372,1		3349	2977	2605	2233			,00401,7		3615	3214	2812	2410	
$\bar{z}.54$	$\bar{1},00293$		0373	0747	1120	1493	1867	$\bar{z}.79$	,09972		0403	0806	1209	1612	2016
	,00373,3		3360	2986	2613	2240			,00403,1		3628	3225	2822	2419	
$\bar{z}.55$	,00666		0374	0749	1123	1497	1872	$\bar{z}.80$	,10376		0404	0809	1213	1618	2022
	,00374,3		3369	2994	2620	2246			,00404,4		3640	3235	2831	2426	
$\bar{z}.56$	,01040		0376	0751	1127	1502	1878	$\bar{z}.81$	,10780		0406	0811	1217	1623	2029
	,00375,5		3380	3004	2629	2253			,00405,7		3651	3246	2840	2434	
$\bar{z}.57$	,01416		0376	0753	1129	1506	1882	$\bar{z}.82$	,11186		0407	0814	1221	1628	2035
	,00376,4		3388	3011	2635	2258			,00407,0		3663	3256	2849	2442	
$\bar{z}.58$	,00792		0378	0756	1134	1512	1890	$\bar{z}.83$	,11593		0408	0817	1225	1633	2042
	,00377,9		3401	3023	2645	2267			,00408,3		3675	3266	2858	2450	
$\bar{z}.59$	,02170		0379	0758	1136	1515	1894	$\bar{z}.84$	,12001		0410	0819	1229	1638	2048
	,00378,3		3409	3030	2652	2273			,00409,5		3686	3276	2867	2457	
$\bar{z}.60$	,02549		0380	0760	1140	1520	1900	$\bar{z}.85$	,12410		0411	0822	1233	1644	2055
	,00380,0		3420	3040	2660	2280			,00410,9		3698	3287	2876	2465	
$\bar{z}.61$	,02929		0381	0762	1144	1525	1906	$\bar{z}.86$	,12821		0412	0824	1237	1649	2061
	,00381,2		3431	3050	2668	2287			,00412,2		3710	3298	2885	2473	
$\bar{z}.62$	,03310		0382	0765	1147	1530	1912	$\bar{z}.87$	,13234		0414	0827	1241	1654	2068
	,00382,4		3442	3059	2677	2294			,00413,6		3722	3309	2895	2482	
$\bar{z}.63$	,03692		0383	0767	1150	1534	1917	$\bar{z}.88$	,13647		0415	0830	1245	1660	2075
	,00383,4		3451	3067	2684	2300			,00414,9		3734	3319	2904	2489	
$\bar{z}.64$	,04076		0385	0769	1154	1539	1924	$\bar{z}.89$	,14062		0416	0833	1249	1665	2082
	,00384,7		3462	3078	2693	2308			,00416,3		3747	3330	2914	2498	
$\bar{z}.65$	,04460		0386	0772	1157	1543	1929	$\bar{z}.90$	,14478		0418	0835	1253	1670	2088
	,00385,8		3472	3086	2701	2315			,00417,6		3758	3341	2923	2506	
$\bar{z}.66$	,04846		0387	0774	1161	1548	1936	$\bar{z}.91$	,14896		0419	0838	1257	1676	2095
	,00387,1		3484	3097	2710	2323			,00419,0		3771	3352	2933	2514	
$\bar{z}.67$	,05233		0388	0776	1165	1553	1941	$\bar{z}.92$	,15315		0420	0841	1261	1682	2102
	,00388,2		3494	3106	2717	2329			,00420,4		3784	3363	2943	2522	
$\bar{z}.68$	,05621		0389	0779	1168	1558	1947	$\bar{z}.93$	,15735		0422	0843	1265	1687	2109
	,00389,4		3505	3115	2726	2336			,00421,7		3795	3344	2952	2530	
$\bar{z}.69$	,06010		0391	0781	1172	1562	1953	$\bar{z}.94$	,16157		0423	0846	1269	1692	2116
	,00390,6		3515	3125	2734	2344			,00423,1		3808	3385	2962	2539	
$\bar{z}.70$	,06401		0392	0784	1176	1568	1960	$\bar{z}.95$	,16580		0425	0849	1274	1698	2123
	,00391,9		3527	3135	2743	2351			,00424,6		3821	3397	2972	2548	
$\bar{z}.71$	,06793		0393	0786	1179	1572	1965	$\bar{z}.96$	,17005		0426	0852	1277	1703	2129
	,00393,0		3537	3144	2751	2358			,00425,8		3832	3406	2981	2555	
$\bar{z}.72$	,07186		0394	0789	1183	1577	1972	$\bar{z}.97$	,17430		0427	0855	1282	1719	2137
	,00394,3		3549	3154	2760	2366			,00427,3		3846	3418	2991	2564	
$\bar{z}.73$	,07581		0396	0791	1187	1582	1978	$\bar{z}.98$	,17858		0429	0857	1286	1715	2144
	,00395,5		3560	3164	2769	2373			,00428,7		3858	3430	3001	2572	
$\bar{z}.74$	,07976		0397	0794	1190	1587	1984	$\bar{z}.99$	,18286		0430	0860	1290	1720	2151
	,00396,8		3571	3174	2778	2381			,00430,1		3871	3441	3011	2581	

General Table III.  $\lambda(a^5), \lambda\left(\frac{1-a^5}{a^5-1}\right)$ .

$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a^5-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a^5-1}\right)$	1 9	2 8	3 7	4 6	5
$\bar{1}.00$	,18717 ,00431,6	,00 0432 3884	0863 3453	1295 3021	1726 2590	2158	$\bar{1}.25$	,29950 ,00469,3	,00 0469 4224	0939 3754	1408 3285	1877 2816	2347
$\bar{1}.01$	,19148 ,00433,0	0433 3897	0866 3464	1299 3031	1732 2598	2165	$\bar{1}.26$	,30419 ,00470,9	0471 4238	0942 3767	1413 3206	1884 2825	2355
$\bar{1}.02$	,19581 ,00434,5	0434 3911	0869 3476	1304 3042	1738 2607	2175	$\bar{1}.27$	,30890 ,00472,5	0473 4253	0945 3780	1418 3308	1890 2835	2363
$\bar{1}.03$	,20016 ,00435,4	0435 3919	0871 3483	1306 3048	1742 2612	2177	$\bar{1}.28$	,31363 ,00474,1	0474 4267	0948 3793	1422 3319	1896 2845	2371
$\bar{1}.04$	,20451 ,00437,6	0438 3938	0875 3401	1313 3063	1750 2626	2188	$\bar{1}.29$	,31837 ,00475,7	0476 4281	0951 3806	1427 3330	1903 2854	2379
$\bar{1}.05$	,20889 ,00438,7	0439 3948	0877 3510	1316 3071	1755 2632	2194	$\bar{1}.30$	,32312 ,00477,3	0477 4296	0955 3818	1432 3341	1909 2864	2387
$\bar{1}.06$	,21328 ,00440,2	0440 3962	0880 3522	1321 3081	1761 2641	2201	$\bar{1}.31$	,32790 ,00479,0	0479 4310	0958 3831	1437 3352	1916 2873	2395
$\bar{1}.07$	,21768 ,00441,7	0442 3975	0883 3534	1325 3092	1767 2650	2209	$\bar{1}.32$	,33269 ,00480,6	0481 4325	0961 3845	1442 3364	1922 2884	2403
$\bar{1}.08$	,22209 ,00443,2	0443 3989	0886 3546	1330 3102	1773 2659	2216	$\bar{1}.33$	,33749 ,00482,2	0482 4340	0964 3858	1447 3375	1929 2893	2411
$\bar{1}.09$	,22653 ,00444,7	0445 4002	0889 3558	1334 3113	1779 2668	2224	$\bar{1}.34$	,34232 ,00483,9	0484 4355	0968 3871	1452 3387	1936 2903	2420
$\bar{1}.10$	,23097 ,00446,1	0446 4015	0892 3569	1338 3123	1784 2677	2231	$\bar{1}.35$	,34715 ,00485,5	0486 4370	0971 3884	1457 3399	1942 2913	2428
$\bar{1}.11$	,23543 ,00447,7	0448 4029	0895 3582	1343 3134	1791 2686	2240	$\bar{1}.36$	,35201 ,00487,2	0487 4385	0974 3898	1462 3410	1949 2923	2436
$\bar{1}.12$	,23991 ,00449,2	0449 4043	0898 3594	1348 3144	1797 2695	2246	$\bar{1}.37$	,35688 ,00488,8	0489 4399	0978 3910	1466 3422	1955 2933	2444
$\bar{1}.13$	,24440 ,00450,9	0451 4058	0902 3607	1353 3156	1804 2705	2255	$\bar{1}.38$	,36177 ,00490,4	0490 4414	0981 3923	1471 3433	1962 2942	2452
$\bar{1}.14$	,24891 ,00452,0	0452 4068	0904 3616	1356 3164	1808 2712	2260	$\bar{1}.39$	,36676 ,00492,1	0492 4429	0984 3937	1476 3445	1968 2953	2461
$\bar{1}.15$	,25343 ,00453,7	0454 4083	0907 3630	1361 3176	1815 2722	2269	$\bar{1}.40$	,37159 ,00493,8	0494 4444	0988 3950	1481 3457	1975 2963	2469
$\bar{1}.16$	,25797 ,00455,3	0455 4098	091 3642	1366 3187	1821 2732	2277	$\bar{1}.41$	,37653 ,00495,5	0496 4460	0991 3964	1487 3469	1982 2973	2477
$\bar{1}.17$	,26252 ,00456,8	0457 4111	0914 3654	1370 3198	1827 2741	2284	$\bar{1}.42$	,38149 ,00497,2	0497 4475	0994 3988	1492 3480	1989 2983	2486
$\bar{1}.18$	,26709 ,00458,3	0458 4125	0917 3666	1375 3208	1833 2750	2292	$\bar{1}.43$	,38646 ,00498,9	0499 4490	0998 3991	1497 3492	1996 2993	2495
$\bar{1}.19$	,27167 ,00459,9	0460 4139	0920 3679	1380 3219	1840 2759	2300	$\bar{1}.44$	,39145 ,00500,6	0501 4505	1001 4005	1502 3504	2002 3004	2503
$\bar{1}.20$	,27627 ,00461,4	0461 4153	0923 3691	1384 3230	1846 2768	2307	$\bar{1}.45$	,39645 ,00502,2	0502 4520	1004 4018	1507 3515	2009 3013	2511
$\bar{1}.21$	,28089 ,00463,0	0463 4167	0926 3704	1389 3241	1852 2778	2315	$\bar{1}.46$	,40148 ,00504,0	0504 4536	1008 4032	1512 3528	2016 3024	2520
$\bar{1}.22$	,28551 ,00464,6	0465 4181	0929 3717	1394 3252	1858 2788	2323	$\bar{1}.47$	,40652 ,00505,7	0506 4551	1011 4046	1517 3540	2023 3034	2529
$\bar{1}.23$	,29016 ,00466,1	0466 4194	0932 3729	1398 3263	1864 2797	2331	$\bar{1}.48$	,41157 ,00507,4	0507 4567	1015 4059	1522 3552	2030 3044	2537
$\bar{1}.24$	,29482 ,00467,7	0468 4209	0935 3742	1403 3274	1871 2806	2339	$\bar{1}.49$	,41665 ,00509,0	0509 4582	1018 4073	1527 3564	2036 3055	2546

General Table III.  $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$ .

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	$\lambda(a^x)$	1 9	2 8	3 7	4 6	5
$\bar{1}.50$	.42174 .00510,8	.00	0511 4597	1022 4086	1532 3576	2043 3065	2554	$\bar{1}.75$	.55471 .00555,2	.00	0555 4997	1110 4442	1666 3886	2221 3331	2776
$\bar{1}.51$	.42684 .00512,5		0513 4613	1025 4100	1538 3588	2050 3075	2563	$\bar{1}.76$	.56026 .00556,9		0557 5012	1114 4455	1671 3898	2228 3341	2785
$\bar{1}.52$	.43197 .00514,2		0514 4628	1028 4114	1543 3599	2057 3085	2571	$\bar{1}.77$	.56583 .00558,6		0559 5027	1117 4469	1676 3910	2234 3352	2793
$\bar{1}.53$	.43711 .00516,1		0516 4645	1032 4129	1548 3613	2064 3097	2581	$\bar{1}.78$	.57142 .00560,7		0561 5046	1121 4486	1682 3925	2243 3364	2804
$\bar{1}.54$	.44227 .00517,8		0518 4660	1036 4142	1553 3625	2071 3107	2589	$\bar{1}.79$	.57702 .00562,4		0562 5062	1125 4499	1687 3937	2250 3374	2812
$\bar{1}.55$	.44745 .00519,5		0520 4676	1039 4156	1559 3637	2078 3117	2598	$\bar{1}.80$	.58265 .00564,2		0564 5078	1128 4514	1693 3949	2257 3385	2821
$\bar{1}.56$	.45264 .00521,2		0521 4691	1042 4170	1564 3648	2085 3127	2606	$\bar{1}.81$	.58829 .00565,9		0566 5093	1132 4527	1698 3961	2264 3395	2830
$\bar{1}.57$	.45786 .00523,0		0523 4707	1046 4184	1569 3661	2092 3138	2615	$\bar{1}.82$	.59395 .00567,8		0568 5110	1136 4542	1703 3975	2271 3407	2839
$\bar{1}.58$	.46309 .00524,7		0525 4722	1049 4198	1574 3673	2099 3148	2624	$\bar{1}.83$	.59963 .00569,7		0570 5127	1139 4558	1709 3988	2279 3418	2849
$\bar{1}.59$	.46833 .00526,5		0527 4739	1053 4212	1580 3686	2106 3159	2633	$\bar{1}.84$	.60532 .00571,5		0572 5144	1143 4572	1715 4001	2286 3429	2858
$\bar{1}.60$	.47350 .00528,3		0528 4755	1057 4226	1585 3698	2113 3170	2642	$\bar{1}.85$	.61104 .00573,3		0573 5160	1147 4586	1720 4013	2293 3440	2867
$\bar{1}.61$	.47888 .00530,0		0530 4770	1060 4240	1590 3710	2120 3180	2650	$\bar{1}.86$	.61677 .00575,1		0575 4176	1150 4601	1725 4026	2300 3451	2876
$\bar{1}.62$	.48418 .00531,8		0532 4786	1064 4254	1595 3723	2127 3191	2659	$\bar{1}.87$	.62252 .00577,0		0577 5193	1154 4616	1731 4039	2308 3462	2885
$\bar{1}.63$	.48950 .00533,6		0534 4802	1067 4269	1601 3735	2134 3202	2668	$\bar{1}.88$	.62829 .00578,8		0579 5210	1158 4630	1736 4052	2315 3473	2894
$\bar{1}.64$	.49484 .00535,3		0535 4818	1071 4282	1606 3747	2141 3212	2677	$\bar{1}.89$	.63408 .00580,8		0581 5227	1162 4646	1742 4066	2323 3485	2904
$\bar{1}.65$	.50019 .00537,1		0537 4834	1074 4297	1611 3760	2148 3223	2688	$\bar{1}.90$	.63989 .00582,5		0583 5243	1165 4660	1748 4078	2330 3495	2913
$\bar{1}.66$	.50556 .00538,9		0539 4850	1078 4311	1617 3772	2156 3233	2695	$\bar{1}.91$	.64572 .00584,4		0584 5260	1169 4675	1753 4091	2338 3506	2922
$\bar{1}.67$	.51095 .00540,7		0541 4866	1081 4326	1622 3785	2163 3244	2704	$\bar{1}.92$	.65156 .00586,2		0586 5276	1172 4690	1759 4103	2345 3517	2931
$\bar{1}.68$	.51636 .00542,5		0543 4883	1085 4340	1628 3798	2170 3255	2713	$\bar{1}.93$	.65742 .00587,9		0588 5291	1176 4713	1764 4115	2352 3527	2940
$\bar{1}.69$	.52178 .00544,3		0544 4899	1089 4354	1633 3810	2177 3266	2722	$\bar{1}.94$	.66330 .00590,0		0590 5310	1180 4720	1770 4130	2360 3540	2950
$\bar{1}.70$	.52722 .00546,1		0546 4915	1092 4369	1638 3823	2184 3277	2731	$\bar{1}.95$	.66920 .00591,7		0592 5325	1183 4734	1775 4142	2367 3550	2959
$\bar{1}.71$	.53269 .00547,8		0548 4920	1096 4382	1643 3835	2191 3287	2739	$\bar{1}.96$	.67512 .00593,5		0594 5342	1187 4748	1781 4155	2374 3561	2968
$\bar{1}.72$	.53816 .00549,7		0550 4947	1099 4398	1649 3848	2199 3298	2749	$\bar{1}.97$	.68105 .00595,4		0595 5359	1191 4763	1786 4168	2382 3572	2977
$\bar{1}.73$	.54366 .00551,5		0552 4964	1103 4412	1655 3861	2206 3309	2758	$\bar{1}.98$	.68701 .00597,3		0597 5376	1195 4778	1791 4181	2389 3584	2987
$\bar{1}.74$	.54918 .00553,3		0553 4980	1107 4426	1660 3873	2213 3320	2767	$\bar{1}.99$	.69298						

General Table IV. For the whole of life. —  $\lambda(a-1)$ .

$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$	$\lambda(a)$	$-\lambda(a-1)$
$\bar{1}.700$	,00206 ,00201	$\bar{1}.725$	,05372 ,00213	$\bar{1}.750$	,10886 ,00229	$\bar{1}.775$	,16826 ,00247	$\bar{1}.800$	,23292 ,00271	$\bar{1}.825$	,30431 ,00302
$\bar{1}.701$	,00407 ,00201	$\bar{1}.726$	,05585 ,00214	$\bar{1}.751$	,11115 ,00230	$\bar{1}.776$	,17073 ,00248	$\bar{1}.801$	,23564 ,00272	$\bar{1}.826$	,30733 ,00304
$\bar{1}.702$	,00608 ,00202	$\bar{1}.727$	,05799 ,00215	$\bar{1}.752$	,11345 ,00230	$\bar{1}.777$	,17322 ,00249	$\bar{1}.802$	,23836 ,00274	$\bar{1}.827$	,31037 ,00305
$\bar{1}.703$	,00810 ,00202	$\bar{1}.728$	,06014 ,00215	$\bar{1}.753$	,11575 ,00231	$\bar{1}.778$	,17571 ,00250	$\bar{1}.803$	,24110 ,00275	$\bar{1}.828$	,31342 ,00307
$\bar{1}.704$	,01012 ,00203	$\bar{1}.729$	,06229 ,00216	$\bar{1}.754$	,11806 ,00232	$\bar{1}.779$	,17822 ,00251	$\bar{1}.804$	,24385 ,00270	$\bar{1}.829$	,31649 ,00308
$\bar{1}.705$	,01214 ,00203	$\bar{1}.730$	,06445 ,00216	$\bar{1}.755$	,12037 ,00233	$\bar{1}.780$	,18073 ,00252	$\bar{1}.805$	,24661 ,00277	$\bar{1}.830$	,31957 ,00309
$\bar{1}.706$	,01418 ,00203	$\bar{1}.731$	,06661 ,00217	$\bar{1}.756$	,12270 ,00233	$\bar{1}.781$	,18325 ,00253	$\bar{1}.806$	,24938 ,00278	$\bar{1}.831$	,32266 ,00311
$\bar{1}.707$	,01621 ,00204	$\bar{1}.732$	,06878 ,00217	$\bar{1}.757$	,12503 ,00234	$\bar{1}.782$	,18578 ,00254	$\bar{1}.807$	,25216 ,00279	$\bar{1}.832$	,32577 ,00313
$\bar{1}.708$	,01825 ,00205	$\bar{1}.733$	,07095 ,00219	$\bar{1}.758$	,12736 ,00235	$\bar{1}.783$	,18832 ,00254	$\bar{1}.808$	,25495 ,00281	$\bar{1}.833$	,32890 ,00314
$\bar{1}.709$	,02030 ,00205	$\bar{1}.734$	,07314 ,00218	$\bar{1}.759$	,12971 ,00235	$\bar{1}.784$	,19086 ,00256	$\bar{1}.809$	,25776 ,00281	$\bar{1}.834$	,33204 ,00315
$\bar{1}.710$	,02235 ,00205	$\bar{1}.735$	,07532 ,00219	$\bar{1}.760$	,13206 ,00236	$\bar{1}.785$	,19342 ,00257	$\bar{1}.810$	,26057 ,00283	$\bar{1}.835$	,33519 ,00317
$\bar{1}.711$	,02440 ,00206	$\bar{1}.736$	,07751 ,00220	$\bar{1}.761$	,13442 ,00237	$\bar{1}.786$	,19599 ,00258	$\bar{1}.811$	,26340 ,00284	$\bar{1}.836$	,33836 ,00319
$\bar{1}.712$	,02646 ,00207	$\bar{1}.737$	,07971 ,00221	$\bar{1}.762$	,13679 ,00237	$\bar{1}.787$	,19856 ,00259	$\bar{1}.812$	,26624 ,00285	$\bar{1}.837$	,34155 ,00320
$\bar{1}.713$	,02853 ,00207	$\bar{1}.738$	,08192 ,00221	$\bar{1}.763$	,13916 ,00238	$\bar{1}.788$	,20115 ,00259	$\bar{1}.813$	,26909 ,00287	$\bar{1}.838$	,34475 ,00322
$\bar{1}.714$	,03060 ,00207	$\bar{1}.739$	,08413 ,00221	$\bar{1}.764$	,14154 ,00239	$\bar{1}.789$	,20374 ,00260	$\bar{1}.814$	,27196 ,00287	$\bar{1}.839$	,34797 ,00324
$\bar{1}.715$	,03267 ,00208	$\bar{1}.740$	,08634 ,00223	$\bar{1}.765$	,14393 ,00240	$\bar{1}.790$	,20634 ,00261	$\bar{1}.815$	,27483 ,00289	$\bar{1}.840$	,35121 ,00325
$\bar{1}.716$	,03476 ,00208	$\bar{1}.741$	,08857 ,00223	$\bar{1}.766$	,14633 ,00240	$\bar{1}.791$	,20896 ,00262	$\bar{1}.816$	,27772 ,00291	$\bar{1}.841$	,35446 ,00327
$\bar{1}.717$	,03684 ,00209	$\bar{1}.742$	,09080 ,00223	$\bar{1}.767$	,14873 ,00241	$\bar{1}.792$	,21158 ,00263	$\bar{1}.817$	,28063 ,00291	$\bar{1}.842$	,35773 ,00329
$\bar{1}.718$	,03893 ,00210	$\bar{1}.743$	,09303 ,00224	$\bar{1}.768$	,15114 ,00242	$\bar{1}.793$	,21421 ,00264	$\bar{1}.818$	,28354 ,00293	$\bar{1}.843$	,36102 ,00331
$\bar{1}.719$	,04103 ,00210	$\bar{1}.744$	,09527 ,00225	$\bar{1}.769$	,15356 ,00243	$\bar{1}.794$	,21685 ,00265	$\bar{1}.819$	,28647 ,00294	$\bar{1}.844$	,36433 ,00332
$\bar{1}.720$	,04313 ,00211	$\bar{1}.745$	,09752 ,00226	$\bar{1}.770$	,15599 ,00244	$\bar{1}.795$	,21951 ,00266	$\bar{1}.820$	,28941 ,00295	$\bar{1}.845$	,36765 ,00334
$\bar{1}.721$	,04524 ,00211	$\bar{1}.746$	,09978 ,00226	$\bar{1}.771$	,15843 ,00244	$\bar{1}.796$	,22217 ,00267	$\bar{1}.821$	,29236 ,00297	$\bar{1}.846$	,37099 ,00336
$\bar{1}.722$	,04735 ,00212	$\bar{1}.747$	,10204 ,00227	$\bar{1}.772$	,16087 ,00245	$\bar{1}.797$	,22484 ,00268	$\bar{1}.822$	,29533 ,00298	$\bar{1}.847$	,37435 ,00338
$\bar{1}.723$	,04947 ,00212	$\bar{1}.748$	,10431 ,00227	$\bar{1}.773$	,16333 ,00246	$\bar{1}.798$	,22753 ,00269	$\bar{1}.823$	,29831 ,00299	$\bar{1}.848$	,37773 ,00339
$\bar{1}.724$	,05159 ,00213	$\bar{1}.749$	,10658 ,00228	$\bar{1}.774$	,16579 ,00247	$\bar{1}.799$	,23022 ,00270	$\bar{1}.824$	,30130 ,00301	$\bar{1}.849$	,38112 ,00342

General Table IV. For the whole of life. —  $\lambda(a^{-1}1)$ .

$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$
$\bar{1}.850$	,38454 ,00343	$\bar{1}.875$	,47688 ,00401	$\bar{1}.900$	,58683 ,00488	$\bar{1}.925$	,72468 ,00635	$\bar{1}.950$	,91357 ,00929	$\bar{1}.975$	1,22728 ,01824
$\bar{1}.851$	,38797 ,00345	$\bar{1}.876$	,48088 ,00405	$\bar{1}.901$	,59171 ,00493	$\bar{1}.926$	,73103 ,00642	$\bar{1}.951$	,92286 ,00946	$\bar{1}.976$	1,24552 ,01899
$\bar{1}.852$	,39142 ,00348	$\bar{1}.877$	,48493 ,00407	$\bar{1}.902$	,59664 ,00497	$\bar{1}.927$	,73745 ,00650	$\bar{1}.952$	,93232 ,00966	$\bar{1}.977$	1,26451 ,01980
$\bar{1}.853$	,39490 ,00349	$\bar{1}.878$	,48900 ,00410	$\bar{1}.903$	,60161 ,00502	$\bar{1}.928$	,74395 ,00659	$\bar{1}.953$	,94198 ,00984	$\bar{1}.978$	1,28431 ,02071
$\bar{1}.854$	,39839 ,00351	$\bar{1}.879$	,49310 ,00412	$\bar{1}.904$	,60663 ,00507	$\bar{1}.929$	,75054 ,00668	$\bar{1}.954$	,95182 ,01006	$\bar{1}.979$	1,30502 ,02170
$\bar{1}.855$	,40190 ,00353	$\bar{1}.880$	,49722 ,00416	$\bar{1}.905$	,61170 ,00511	$\bar{1}.930$	,75722 ,00676	$\bar{1}.955$	,96188 ,01027	$\bar{1}.980$	1,32672 ,02278
$\bar{1}.856$	,40543 ,00355	$\bar{1}.881$	,50138 ,00419	$\bar{1}.906$	,61681 ,00516	$\bar{1}.931$	,76398 ,00685	$\bar{1}.956$	,97215 ,01049	$\bar{1}.981$	1,34950 ,02398
$\bar{1}.857$	,40899 ,00357	$\bar{1}.882$	,50557 ,00422	$\bar{1}.907$	,62197 ,00521	$\bar{1}.932$	,77083 ,00695	$\bar{1}.957$	,98264 ,01073	$\bar{1}.982$	1,37348 ,02583
$\bar{1}.858$	,41256 ,00360	$\bar{1}.883$	,50979 ,00425	$\bar{1}.908$	,62718 ,00527	$\bar{1}.933$	,77778 ,00704	$\bar{1}.958$	,99337 ,01097	$\bar{1}.983$	1,39881 ,02683
$\bar{1}.859$	,41616 ,00362	$\bar{1}.884$	,51404 ,00428	$\bar{1}.909$	,63245 ,00531	$\bar{1}.934$	,78482 ,00715	$\bar{1}.959$	1,00434 ,01123	$\bar{1}.984$	1,42564 ,02853
$\bar{1}.860$	,41978 ,00364	$\bar{1}.885$	,51832 ,00431	$\bar{1}.910$	,63776 ,00537	$\bar{1}.935$	,79197 ,00724	$\bar{1}.960$	1,01557 ,01150	$\bar{1}.985$	1,45417 ,03047
$\bar{1}.861$	,42342 ,00366	$\bar{1}.886$	,52263 ,00435	$\bar{1}.911$	,64313 ,00543	$\bar{1}.936$	,79921 ,00735	$\bar{1}.961$	1,02707 ,01179	$\bar{1}.986$	1,48464 ,03269
$\bar{1}.862$	,42708 ,00368	$\bar{1}.887$	,52698 ,00438	$\bar{1}.912$	,64856 ,00548	$\bar{1}.937$	,80656 ,00746	$\bar{1}.962$	1,03886 ,01209	$\bar{1}.987$	1,51733 ,03526
$\bar{1}.863$	,44076 ,00371	$\bar{1}.888$	,53136 ,00442	$\bar{1}.913$	,65404 ,00554	$\bar{1}.938$	,81402 ,00758	$\bar{1}.963$	1,05095 ,01241	$\bar{1}.988$	1,55259 ,03829
$\bar{1}.864$	,43447 ,00373	$\bar{1}.889$	,53578 ,00445	$\bar{1}.914$	,65958 ,00559	$\bar{1}.939$	,82160 ,00769	$\bar{1}.964$	1,06336 ,01274	$\bar{1}.989$	1,59088 ,04189
$\bar{1}.865$	,43820 ,00376	$\bar{1}.890$	,54023 ,00449	$\bar{1}.915$	,66517 ,00566	$\bar{1}.940$	,82929 ,00781	$\bar{1}.965$	1,07610 ,01309	$\bar{1}.990$	1,63277 ,04626
$\bar{1}.866$	,44196 ,00378	$\bar{1}.891$	,54472 ,00452	$\bar{1}.916$	,67083 ,00572	$\bar{1}.941$	,83710 ,00793	$\bar{1}.966$	1,08919 ,01348	$\bar{1}.991$	1,67903 ,05163
$\bar{1}.867$	,44574 ,00380	$\bar{1}.892$	,54924 ,00456	$\bar{1}.917$	,67655 ,00578	$\bar{1}.942$	,84503 ,00807	$\bar{1}.967$	1,10267 ,01387	$\bar{1}.992$	1,73069 ,05849
$\bar{1}.868$	,44954 ,00383	$\bar{1}.893$	,55380 ,00460	$\bar{1}.918$	,68233 ,00584	$\bar{1}.943$	,85310 ,00820	$\bar{1}.968$	1,11654 ,01429	$\bar{1}.993$	1,78918 ,06745
$\bar{1}.869$	,45337 ,00385	$\bar{1}.894$	,55840 ,00464	$\bar{1}.919$	,68817 ,00591	$\bar{1}.944$	,86130 ,00833	$\bar{1}.969$	1,13083 ,01475	$\bar{1}.994$	1,85663 ,07968
$\bar{1}.870$	,45722 ,00388	$\bar{1}.895$	,56304 ,00467	$\bar{1}.920$	,69408 ,00598	$\bar{1}.945$	,86963 ,00848	$\bar{1}.970$	1,14558 ,01523	$\bar{1}.995$	1,93631 ,09741
$\bar{1}.871$	,46110 ,00390	$\bar{1}.896$	,56771 ,00472	$\bar{1}.921$	,70006 ,00695	$\bar{1}.946$	,87811 ,00863	$\bar{1}.971$	1,16081 ,01574	$\bar{1}.996$	2,03372 ,12544
$\bar{1}.872$	,46500 ,00393	$\bar{1}.897$	,57243 ,00476	$\bar{1}.922$	,70611 ,00611	$\bar{1}.947$	,88674 ,00878	$\bar{1}.972$	1,17655 ,01530	$\bar{1}.997$	2,15916 ,17459
$\bar{1}.873$	,46893 ,00396	$\bar{1}.898$	,57719 ,00479	$\bar{1}.923$	,71222 ,00620	$\bar{1}.948$	,89552 ,00894	$\bar{1}.973$	1,19285 ,01690	$\bar{1}.998$	2,33575 ,30153
$\bar{1}.874$	,47289 ,00399	$\bar{1}.899$	,58198 ,00485	$\bar{1}.924$	,71842 ,00626	$\bar{1}.949$	,90446 ,00911	$\bar{1}.974$	1,20975 ,01753	$\bar{1}.999$	2,63728

TABLE V.—Logarithms of the accommodated chances of living 10 years, deduced from the value of an annuity for 10 years, at 5 per cent. from the actual tables of mortality, and considered equal to a geometrical series of ten terms, of which the common ratio is the same as the first term, and the tenth term the accommodated chance; and to find the accommodated chance for 5, 7 years, &c. without a table calculated for the purpose, it may be considered sufficient to multiply by .5; .7, &c. the accommodated ratio in this table when extreme accuracy be not required.

Age.	Carlisle.	Deparcieux.	Northampton.	Age.	Carlisle.	Deparcieux.	Northampton.
0	1.6892	—	—	52	1.9172	1.9006	1.8523
1	1.6763	—	1.7044	53	1.9098	1.8957	1.8471
2	1.8699	—	1.8356	54	1.9013	1.8901	1.8417
3	1.9159	1.9166	1.8790	55	1.8915	1.8853	1.8357
4	1.9401	1.9315	1.9081	56	1.8803	1.8799	1.8294
5	1.9586	1.9411	1.9220	57	1.8680	1.8732	1.8228
6	1.9686	1.9486	1.9369	58	1.8513	1.8673	1.8156
7	1.9737	1.9544	1.9476	59	1.8435	1.8601	1.8081
8	1.9764	1.9592	1.9550	60	1.8318	1.8511	1.7998
9	1.9773	1.9637	1.9586	61	1.8243	1.8398	1.7908
10	1.9768	1.9669	1.9592	62	1.8171	1.8264	1.7811
11	1.9754	1.9679	1.9582	63	1.8090	1.8120	1.7699
12	1.9742	1.9669	1.9566	64	1.7974	1.7946	1.7576
13	1.9729	1.9658	1.9546	65	1.7860	1.7735	1.7431
14	1.9716	1.9704	1.9521	66	1.7703	1.7510	1.7267
15	1.9704	1.9628	1.9490	67	1.7506	1.7270	1.7083
16	1.9698	1.9609	1.9455	68	1.7107	1.7017	1.6879
17	1.9694	1.9600	1.9419	69	1.7005	1.6754	1.6651
18	1.9693	1.9586	1.9388	70	1.6689	1.6480	1.6402
19	1.9690	1.9574	1.9358	71	1.6319	1.6167	1.6126
20	1.9685	1.9559	1.9337	72	1.5936	1.5841	1.5823
21	1.9679	1.9554	1.9321	73	1.5563	1.5500	1.5487
22	1.9670	1.9549	1.9311	74	1.5269	1.5119	1.5117
23	1.9659	1.9544	1.9298	75	1.4940	1.4711	1.4723
24	1.9644	1.9540	1.9289	76	1.4642	1.4218	1.4308
25	1.9628	1.9534	1.9277	77	1.4344	1.3684	1.3846
26	1.9573	1.9531	1.9264	78	1.4007	1.3134	1.3307
27	1.9591	1.9524	1.9257	79	1.3538	1.2497	1.2644
28	1.9570	1.9521	1.9238	80	1.3134	1.1876	1.1900
29	1.9556	1.9518	1.9226	81	1.2582	1.1214	1.1101
30	1.9552	1.9514	1.9211	82	1.2043	1.0609	1.0234
31	1.9548	1.9514	1.9196	83	1.1765	2.9688	2.9341
32	1.9540	1.9514	1.9180	84	1.0727	2.8536	2.8592
33	1.9528	1.9515	1.9164	85	2.9939	2.7199	2.7813
34	1.9513	1.9517	1.9146	86	2.9166	2.5736	2.7003
35	1.9485	1.9522	1.9126	87	2.8490	2.4254	2.6149
36	1.9477	1.9528	1.9104	88	2.8055	2.1943	2.5369
37	1.9452	1.9534	1.9083	89	2.7537	3.9129	2.4179
38	1.9437	1.9527	1.8057	90	2.6695	3.5265	2.2414
39	1.9406	1.9517	1.9031	91	2.6658	3.0266	3.9356
40	1.9383	1.9506	1.9001	92	2.7323	4.3694	3.5037
41	1.9372	1.9488	1.8973	93	2.8031	5.2971	4.7375
42	1.9365	1.9466	1.8943	94	2.8355	—	5.5769
43	1.9365	1.9438	1.8915	95	2.8107	—	7.0496
44	1.9366	1.9403	1.8882	96	2.8279	—	—
45	1.9367	1.9361	1.8848	97	2.7589	—	—
46	1.9366	1.9308	1.8810	98	2.6695	—	—
47	1.9358	1.9263	1.8767	99	2.5111	—	—
48	1.9351	1.9200	1.8740	100	2.1629	—	—
49	1.9328	1.9158	1.8631	101	3.5689	—	—
50	1.9292	1.9098	1.8621	102	4.3245	—	—
51	1.9233	1.9027	1.8571	103	6.0595	—	—

TABLE VI.—Accommodated annual ratio for an unlimited period for every age  $a$

$$\lambda r = \lambda \overset{1.05^{-1}}{i} a - \lambda \overset{1.05^{-1}}{0} a + \lambda 1.05.$$

$a$	$\lambda r$ Carlisle.	$\lambda r$ Deparcieux.	$\lambda r$ Northampton.	$a$	$\lambda r$ Carlisle.	$\lambda r$ Deparcieux.	$\lambda r$ Northampton.
0	1.98665	—	—	52	1.98390	1.98216	1.97950
1	1.99121	—	1.98517	53	1.98305	1.98240	1.97878
2	1.99313	—	1.98997	54	1.98212	1.98156	1.97802
3	1.99458	1.99399	1.99151	55	1.98112	1.98073	1.97721
4	1.99528	1.99446	1.99244	56	1.98005	1.97982	1.97635
5	1.99577	1.99473	1.99284	57	1.97887	1.97882	1.97542
6	1.99599	1.99493	1.99324	58	1.97763	1.97780	1.97445
7	1.99606	1.99505	1.99346	59	1.97637	1.97668	1.97341
8	1.99606	1.99513	1.99357	60	1.97514	1.97545	1.97230
9	1.99600	1.99519	1.99354	61	1.97400	1.97408	1.97111
10	1.99529	1.99519	1.99341	62	1.97281	1.97254	1.96983
11	1.99576	1.99514	1.99323	63	1.97154	1.97093	1.96843
12	1.99563	1.99503	1.99304	64	1.97014	1.96912	1.96693
13	1.99549	1.99491	1.99284	65	1.96858	1.96707	1.96526
14	1.99535	1.99478	1.99262	66	1.96685	1.96489	1.96346
15	1.99522	1.99464	1.99240	67	1.96491	1.96250	1.96150
16	1.99509	1.99450	1.99213	68	1.96273	1.96009	1.95936
17	1.99487	1.99438	1.99190	69	1.96029	1.95750	1.95703
18	1.99484	1.99425	1.99167	70	1.95755	1.95480	1.95448
19	1.99471	1.99413	1.99145	71	1.95440	1.95181	1.95171
20	1.99455	1.99398	1.99124	72	1.95111	1.94870	1.94869
21	1.99442	1.99388	1.99106	73	1.94784	1.94542	1.94541
22	1.99425	1.99375	1.99088	74	1.94467	1.94180	1.94186
23	1.99408	1.99364	1.99070	75	1.94185	1.93790	1.93812
24	1.99390	1.99350	1.99051	76	1.93888	1.93330	1.93423
25	1.99370	1.99338	1.99030	77	1.93591	1.92833	1.92994
26	1.99349	1.99323	1.99009	78	1.93265	1.92314	1.92503
27	1.99328	1.99308	1.98988	79	1.92863	1.91715	1.91916
28	1.99306	1.99293	1.98965	80	1.92461	1.91124	1.91246
29	1.99290	1.99277	1.98943	81	1.92049	1.90493	1.90509
30	1.99265	1.99259	1.98917	82	1.91491	1.89856	1.89697
31	1.99245	1.99241	1.98892	83	1.90939	1.89069	1.88844
32	1.99224	1.99223	1.98865	84	1.90344	1.88005	1.88110
33	1.99201	1.99202	1.98837	85	1.89657	1.86782	1.87361
34	1.99176	1.99181	1.98808	86	1.88978	1.85416	1.86599
35	1.99149	1.99158	1.98777	87	1.88369	1.84011	1.85803
36	1.99110	1.99134	1.98745	88	1.87792	1.81788	1.85069
37	1.99090	1.99109	1.98811	89	1.87481	1.78941	1.85454
38	1.99058	1.99077	1.98675	90	1.86660	1.75223	1.82243
39	1.99025	1.99043	1.98637	91	1.86560	1.70265	1.79303
40	1.98991	1.99006	1.98597	92	1.87056	1.63694	1.74992
41	1.98958	1.98967	1.98556	93	1.87595	1.52971	1.67376
42	1.98924	1.98924	1.98513	94	1.87840	—	1.55753
43	1.98890	1.98878	1.98469	95	1.87967	—	1.30505
44	1.98853	1.98828	1.98423	96	1.77774	—	—
45	1.98814	1.98771	1.98375	97	1.77140	—	—
46	1.98771	1.98714	1.98323	98	1.76333	—	—
47	1.98725	1.98655	1.98270	99	1.84829	—	—
48	1.98673	1.98590	1.98209	100	1.81282	—	—
49	1.98612	1.98526	1.98148	101	1.75663	—	—
50	1.98546	1.98456	1.98083	102	1.65421	—	—
51	1.98471	1.98386	1.98017	103	1.40266	—	—



TABLE VII.—Logarithm of Carlisle chance of living 5 years at every age *a*.

<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.
0	$\bar{1},83232$	20	$\bar{1},98469$	40	$\bar{1},96915$	60	$\bar{1},91826$	80	$\bar{1},66927$
1	$\bar{1},89709$	21	$\bar{1},98457$	41	$\bar{1},96836$	61	$\bar{1},91483$	81	$\bar{1},64194$
2	$\bar{1},92823$	22	$\bar{1},98439$	42	$\bar{1},96790$	62	$\bar{1},91180$	82	$\bar{1},61095$
3	$\bar{1},95354$	23	$\bar{1},98405$	43	$\bar{1},96780$	63	$\bar{1},90864$	83	$\bar{1},57100$
4	$\bar{1},96747$	24	$\bar{1},98333$	44	$\bar{1},96808$	64	$\bar{1},90492$	84	$\bar{1},53422$
5	$\bar{1},97792$	25	$\bar{1},98213$	45	$\bar{1},96857$	65	$\bar{1},90067$	85	$\bar{1},50393$
6	$\bar{1},98376$	26	$\bar{1},98091$	46	$\bar{1},96918$	66	$\bar{1},89586$	86	$\bar{1},45652$
7	$\bar{1},98703$	27	$\bar{1},97967$	47	$\bar{1},96941$	67	$\bar{1},88838$	87	$\bar{1},40377$
8	$\bar{1},98869$	28	$\bar{1},97863$	48	$\bar{1},96915$	68	$\bar{1},87746$	88	$\bar{1},36691$
9	$\bar{1},98930$	29	$\bar{1},97804$	49	$\bar{1},96818$	69	$\bar{1},86279$	89	$\bar{1},34438$
10	$\bar{1},98911$	30	$\bar{1},97789$	50	$\bar{1},96676$	70	$\bar{1},84362$	90	$\bar{1},32483$
11	$\bar{1},98836$	31	$\bar{1},97783$	51	$\bar{1},96477$	71	$\bar{1},82305$	91	$\bar{1},34054$
12	$\bar{1},98754$	32	$\bar{1},97767$	52	$\bar{1},96269$	72	$\bar{1},80220$	92	$\bar{1},38021$
13	$\bar{1},98670$	33	$\bar{1},97736$	53	$\bar{1},96017$	73	$\bar{1},78348$	93	$\bar{1},41373$
14	$\bar{1},98593$	34	$\bar{1},97687$	54	$\bar{1},95660$	74	$\bar{1},76877$	94	$\bar{1},43933$
15	$\bar{1},98528$	35	$\bar{1},97611$	55	$\bar{1},95155$	75	$\bar{1},75508$	95	$\bar{1},47712$
16	$\bar{1},98490$	36	$\bar{1},97490$	56	$\bar{1},94461$	76	$\bar{1},74231$	96	$\bar{1},48337$
17	$\bar{1},98479$	37	$\bar{1},97349$	57	$\bar{1},93711$	77	$\bar{1},72712$	97	$\bar{1},44370$
18	$\bar{1},98476$	38	$\bar{1},97194$	58	$\bar{1},92973$	78	$\bar{1},71062$	98	$\bar{1},33099$
19	$\bar{1},98472$	39	$\bar{1},97044$	59	$\bar{1},92343$	79	$\bar{1},68963$		

Logarithm of the Carlisle chance of living 10 years at every age *a*.

0	$\bar{1},81023$	19	$\bar{1},96805$	38	$\bar{1},93973$	57	$\bar{1},84891$	76	$\bar{1},38425$
1	$\bar{1},88086$	20	$\bar{1},96682$	39	$\bar{1},93851$	58	$\bar{1},83836$	77	$\bar{1},33807$
2	$\bar{1},91526$	21	$\bar{1},96548$	40	$\bar{1},93772$	59	$\bar{1},82835$	78	$\bar{1},28163$
3	$\bar{1},94223$	22	$\bar{1},96406$	41	$\bar{1},93754$	60	$\bar{1},81893$	79	$\bar{1},22385$
4	$\bar{1},95677$	23	$\bar{1},96268$	42	$\bar{1},93731$	61	$\bar{1},81070$	80	$\bar{1},17320$
5	$\bar{1},96702$	24	$\bar{1},96136$	43	$\bar{1},93694$	62	$\bar{1},80018$	81	$\bar{1},09846$
6	$\bar{1},97213$	25	$\bar{1},96002$	44	$\bar{1},93626$	63	$\bar{1},78610$	82	$\bar{1},01472$
7	$\bar{1},97457$	26	$\bar{1},95873$	45	$\bar{1},93533$	64	$\bar{1},76771$	83	$\bar{2},93791$
8	$\bar{1},97540$	27	$\bar{1},95734$	46	$\bar{1},93395$	65	$\bar{1},74430$	84	$\bar{2},87860$
9	$\bar{1},97523$	28	$\bar{1},95598$	47	$\bar{1},93211$	66	$\bar{1},71891$	85	$\bar{2},82876$
10	$\bar{1},97438$	29	$\bar{1},95490$	48	$\bar{1},92932$	67	$\bar{1},69058$	86	$\bar{2},79706$
11	$\bar{1},97326$	30	$\bar{1},95400$	49	$\bar{1},92478$	68	$\bar{1},66094$	87	$\bar{2},78398$
12	$\bar{1},97233$	31	$\bar{1},95273$	50	$\bar{1},91830$	69	$\bar{1},63157$	88	$\bar{2},78064$
13	$\bar{1},97146$	32	$\bar{1},95116$	51	$\bar{1},90938$	70	$\bar{1},59870$	89	$\bar{2},78371$
14	$\bar{1},07065$	33	$\bar{1},94929$	52	$\bar{1},89980$	71	$\bar{1},56536$	90	$\bar{2},80195$
15	$\bar{1},96996$	34	$\bar{1},94730$	53	$\bar{1},88990$	72	$\bar{1},52932$	91	$\bar{2},82391$
16	$\bar{1},96947$	35	$\bar{1},94526$	54	$\bar{1},88003$	73	$\bar{1},49411$	92	$\bar{2},82391$
17	$\bar{1},96918$	36	$\bar{1},94326$	55	$\bar{1},86981$	74	$\bar{1},45840$	93	$\bar{2},74473$
18	$\bar{1},96881$	37	$\bar{1},94138$	56	$\bar{1},85944$	75	$\bar{1},42434$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 15 years for every age *a*.

<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.
0	$\bar{1},79934$	18	$\bar{1},94744$	36	$\bar{1},91244$	54	$\bar{1},78495$	72	$\bar{1},14027$
1	$\bar{1},86922$	19	$\bar{1},94609$	37	$\bar{1},91080$	55	$\bar{1},77048$	73	$\bar{1},06511$
2	$\bar{1},90280$	20	$\bar{1},94471$	38	$\bar{1},90888$	56	$\bar{1},75530$	74	$\bar{2},99263$
3	$\bar{1},92893$	21	$\bar{1},94331$	39	$\bar{1},90669$	57	$\bar{1},73729$	75	$\bar{2},92827$
4	$\bar{1},94270$	22	$\bar{1},94173$	40	$\bar{1},90448$	58	$\bar{1},71582$	76	$\bar{2},84078$
5	$\bar{1},95230$	23	$\bar{1},94004$	41	$\bar{1},90231$	59	$\bar{1},69114$	77	$\bar{2},74184$
6	$\bar{1},95702$	24	$\bar{1},93823$	42	$\bar{1},90000$	60	$\bar{1},66256$	78	$\bar{2},64853$
7	$\bar{1},95936$	25	$\bar{1},93613$	43	$\bar{1},89712$	61	$\bar{1},63375$	79	$\bar{2},56823$
8	$\bar{1},96015$	26	$\bar{1},93364$	44	$\bar{1},89284$	62	$\bar{1},60238$	80	$\bar{2},49803$
9	$\bar{1},95995$	27	$\bar{1},93082$	45	$\bar{1},88687$	63	$\bar{1},56958$	81	$\bar{2},43900$
10	$\bar{1},95907$	28	$\bar{1},92792$	46	$\bar{1},87856$	64	$\bar{1},53648$	82	$\bar{2},39494$
11	$\bar{1},95784$	29	$\bar{1},92534$	47	$\bar{1},86922$	65	$\bar{1},49937$	83	$\bar{2},35164$
12	$\bar{1},95672$	30	$\bar{1},92316$	48	$\bar{1},85905$	66	$\bar{1},46123$	84	$\bar{2},31794$
13	$\bar{1},95551$	31	$\bar{1},92108$	49	$\bar{1},84820$	67	$\bar{1},41770$	85	$\bar{2},30588$
14	$\bar{1},95397$	32	$\bar{1},91905$	50	$\bar{1},83656$	68	$\bar{1},37157$	86	$\bar{2},28043$
15	$\bar{1},95209$	33	$\bar{1},91709$	51	$\bar{1},82421$	69	$\bar{1},32120$	87	$\bar{2},22768$
16	$\bar{1},95038$	34	$\bar{1},91538$	52	$\bar{1},81160$	70	$\bar{1},26797$	88	$\bar{2},11163$
17	$\bar{1},94885$	35	$\bar{1},91383$	53	$\bar{1},79853$	71	$\bar{1},20730$		

Logarithm of the Carlisle chance of living 20 years for every age *a*.

0	$\bar{1},78462$	17	$\bar{1},92652$	34	$\bar{1},88356$	51	$\bar{1},72007$	68	$\bar{2},94257$
1	$\bar{1},85412$	18	$\bar{1},92479$	35	$\bar{1},88059$	52	$\bar{1},69998$	69	$\bar{2},85542$
2	$\bar{1},88759$	19	$\bar{1},92295$	36	$\bar{1},87721$	53	$\bar{1},67599$	70	$\bar{2},77190$
3	$\bar{1},91369$	20	$\bar{1},92082$	37	$\bar{1},87349$	54	$\bar{1},64744$	71	$\bar{2},66383$
4	$\bar{1},92742$	21	$\bar{1},91821$	38	$\bar{1},86906$	55	$\bar{1},61410$	72	$\bar{2},54404$
5	$\bar{1},93699$	22	$\bar{1},91521$	39	$\bar{1},86329$	56	$\bar{1},57835$	73	$\bar{2},43202$
6	$\bar{1},94160$	23	$\bar{1},91197$	40	$\bar{1},85602$	57	$\bar{1},53949$	74	$\bar{2},33701$
7	$\bar{1},94376$	24	$\bar{1},90866$	41	$\bar{1},84692$	58	$\bar{1},49930$	75	$\bar{2},25311$
8	$\bar{1},94421$	25	$\bar{1},90528$	42	$\bar{1},83711$	59	$\bar{1},45991$	76	$\bar{2},18132$
9	$\bar{1},94328$	26	$\bar{1},90199$	43	$\bar{1},82684$	60	$\bar{1},41763$	77	$\bar{2},12205$
10	$\bar{1},94120$	27	$\bar{1},89872$	44	$\bar{1},81628$	61	$\bar{1},37606$	78	$\bar{2},06227$
11	$\bar{1},93874$	28	$\bar{1},89572$	45	$\bar{1},80513$	62	$\bar{1},32950$	79	$\bar{2},00757$
12	$\bar{1},93639$	29	$\bar{1},89342$	46	$\bar{1},79339$	63	$\bar{1},28021$	80	$\bar{3},97515$
13	$\bar{1},93414$	30	$\bar{1},89172$	47	$\bar{1},78101$	64	$\bar{1},22611$	81	$\bar{3},92237$
14	$\bar{1},93201$	31	$\bar{1},89027$	48	$\bar{1},76768$	65	$\bar{1},16864$	82	$\bar{3},83863$
15	$\bar{1},92999$	32	$\bar{1},88847$	49	$\bar{1},75312$	66	$\bar{1},10317$	83	$\bar{3},68263$
16	$\bar{1},92821$	33	$\bar{1},88624$	50	$\bar{1},73723$	67	$\bar{1},02866$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 25 years at every age *a*.

<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.
0	1,76930	16	1,90311	32	1,85116	48	1,64514	64	2,76034
1	1,83969	17	1,90001	33	1,84641	49	1,61591	65	2,67257
2	1,87198	18	1,89673	34	1,84016	50	1,58086	66	2,55969
3	1,89774	19	1,89339	35	1,83213	51	1,5431	67	2,43242
4	1,91075	20	1,88997	36	1,82182	52	1,50218	68	2,30948
5	1,91912	21	1,88657	37	1,81060	53	1,45948	69	2,19980
6	1,92251	22	1,88311	38	1,79878	54	1,41651	70	2,09673
7	1,92342	23	1,87977	39	1,78672	55	1,36918	71	2,00437
8	1,92283	24	1,87674	40	1,77428	56	1,32067	72	1,92425
9	1,92131	25	1,87385	41	1,76175	57	1,26661	73	1,84575
10	1,91909	26	1,87117	42	1,74891	58	1,20993	74	1,77634
11	1,91657	27	1,86813	43	1,73548	59	1,14954	75	1,71323
12	1,91406	28	1,86487	44	1,72120	60	1,08690	76	1,66469
13	1,91150	29	1,86159	45	1,70581	61	1,01800	77	1,61675
14	1,90888	30	1,85848	46	1,68926	62	1,94045	78	1,57326
15	1,90610	31	1,85504	47	1,66940	63	1,85121		

Logarithm of the Carlisle chance of living 30 years at every age *a*.

0	1,75143	15	1,87525	30	1,81003	45	1,54943	60	2,59083
1	1,81960	16	1,87146	31	1,79964	46	1,51231	61	2,47452
2	1,85164	17	1,86790	32	1,78827	47	1,47160	62	2,34422
3	1,87637	18	1,86453	33	1,77614	48	1,42863	63	2,21811
4	1,88879	19	1,86147	34	1,76358	49	1,38467	64	2,10472
5	1,89701	20	1,85854	35	1,75039	50	1,33594	65	1,99740
6	1,90033	21	1,85575	36	1,73665	51	1,28544	66	1,90023
7	1,90109	22	1,85253	37	1,72240	52	1,22930	67	1,81264
8	1,90019	23	1,84892	38	1,70742	53	1,17010	68	1,72321
9	1,89818	24	1,84492	39	1,69164	54	1,10614	69	1,63913
10	1,89520	25	1,84061	40	1,67496	55	1,03845	70	1,55735
11	1,89147	26	1,83594	41	1,65761	56	1,96261	71	1,48774
12	1,88755	27	1,83083	42	1,63729	57	1,87756	72	1,41679
13	1,88344	28	1,82504	43	1,61294	58	1,78093	73	1,34674
14	1,87931	29	1,81819	44	1,58399	59	1,68376		

Logarithm of the Carlisle chance of living 35 years for every age *a*.

0	1,72933	14	1,84739	28	1,75476	42	1,43949	56	2,41913
1	1,79743	15	1,84382	29	1,74162	43	1,39642	57	2,28133
2	1,82932	16	1,84065	30	1,72829	44	1,35277	58	2,14784
3	1,85373	17	1,83732	31	1,71448	45	1,30451	59	2,02815
4	1,86565	18	1,83368	32	1,70007	46	1,25462	60	1,91566
5	1,87312	19	1,82964	33	1,68477	47	1,19872	61	1,81506
6	1,87523	20	1,82530	34	1,66850	48	1,13925	62	1,72443
7	1,87458	21	1,82052	35	1,65106	49	1,07432	63	1,63185
8	1,87213	22	1,81522	36	1,63251	50	1,00520	64	1,54405
9	1,86861	23	1,80909	37	1,61078	51	1,92738	65	1,47452
10	1,86435	24	1,80152	38	1,58488	52	1,84025	66	1,38360
11	1,85983	25	1,79216	39	1,55443	53	1,74110	67	1,29633
12	1,85544	26	1,78055	40	1,51858	54	1,64036	68	1,205420
13	1,85123	27	1,76794	41	1,48066	55	1,54237		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 40 years for every age *a*.

<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.
0	$\bar{1},70544$	13	$\bar{1},82038$	26	$\bar{1},69538$	39	$\bar{1},32320$	52	$\bar{2},24402$
1	$\bar{1},77233$	14	$\bar{1},81557$	27	$\bar{1},67973$	40	$\bar{1},27366$	53	$\bar{2},10801$
2	$\bar{1},80280$	15	$\bar{1},81057$	28	$\bar{1},66340$	41	$\bar{1},22298$	54	$\bar{3},98474$
3	$\bar{1},82567$	16	$\bar{1},80542$	29	$\bar{1},64654$	42	$\bar{1},16661$	55	$\bar{3},86721$
4	$\bar{1},83609$	17	$\bar{1},80001$	30	$\bar{1},62896$	43	$\bar{1},10705$	56	$\bar{3},75967$
5	$\bar{1},84227$	18	$\bar{1},79385$	31	$\bar{1},61034$	44	$\bar{1},04240$	57	$\bar{3},66154$
6	$\bar{1},84359$	19	$\bar{1},78624$	32	$\bar{1},58845$	45	$\bar{2},97377$	58	$\bar{3},56157$
7	$\bar{1},84247$	20	$\bar{1},77684$	33	$\bar{1},56223$	46	$\bar{2},89656$	59	$\bar{3},46748$
8	$\bar{1},83992$	21	$\bar{1},76513$	34	$\bar{1},53130$	47	$\bar{2},80967$	60	$\bar{3},39278$
9	$\bar{1},83669$	22	$\bar{1},75233$	35	$\bar{1},49469$	48	$\bar{2},71025$	61	$\bar{3},29843$
10	$\bar{1},83292$	23	$\bar{1},73882$	36	$\bar{1},45556$	49	$\bar{2},60854$	62	$\bar{3},16813$
11	$\bar{1},82901$	24	$\bar{1},72494$	37	$\bar{1},41298$	50	$\bar{2},50913$	63	$\bar{4},96284$
12	$\bar{1},82486$	25	$\bar{1},71042$	38	$\bar{1},36836$	51	$\bar{2},38390$		

Logarithm of the Carlisle chance of living 45 years for every age *a*.

0	$\bar{1},67459$	12	$\bar{1},78755$	24	$\bar{1},62987$	36	$\bar{1},19787$	48	$\bar{2},07716$
1	$\bar{1},74068$	13	$\bar{1},78055$	25	$\bar{1},61109$	37	$\bar{1},14010$	49	$\bar{3},95292$
2	$\bar{1},77070$	14	$\bar{1},77217$	26	$\bar{1},59125$	38	$\bar{1},07899$	50	$\bar{3},83396$
3	$\bar{1},79346$	15	$\bar{1},76212$	27	$\bar{1},56812$	39	$\bar{1},01283$	51	$\bar{3},72444$
4	$\bar{1},80417$	16	$\bar{1},75002$	28	$\bar{1},54086$	40	$\bar{2},94292$	52	$\bar{3},62423$
5	$\bar{1},81084$	17	$\bar{1},73712$	29	$\bar{1},50933$	41	$\bar{2},86492$	53	$\bar{3},52174$
6	$\bar{1},81277$	18	$\bar{1},72357$	30	$\bar{1},47258$	42	$\bar{2},77756$	54	$\bar{3},42408$
7	$\bar{1},81190$	19	$\bar{1},70967$	31	$\bar{1},43339$	43	$\bar{2},67805$	55	$\bar{3},34433$
8	$\bar{1},80907$	20	$\bar{1},69510$	32	$\bar{1},39065$	44	$\bar{2},57662$	56	$\bar{3},24304$
9	$\bar{1},80487$	21	$\bar{1},67996$	33	$\bar{1},34571$	45	$\bar{2},47770$	57	$\bar{3},10524$
10	$\bar{1},79968$	22	$\bar{1},66412$	34	$\bar{1},30007$	46	$\bar{2},35308$	58	$\bar{4},89256$
11	$\bar{1},79378$	23	$\bar{1},64745$	35	$\bar{1},24977$	47	$\bar{2},21344$		

Logarithm of the Carlisle chance of living 50 years for every age *a*.

0	$\bar{1},64316$	11	$\bar{1},73839$	22	$\bar{1},55251$	33	$\bar{1},05634$	44	$\bar{3},92100$
1	$\bar{1},70987$	12	$\bar{1},72466$	23	$\bar{1},52491$	34	$\bar{2},93970$	45	$\bar{3},80253$
2	$\bar{1},74011$	13	$\bar{1},71028$	24	$\bar{1},49266$	35	$\bar{2},91903$	46	$\bar{3},69362$
3	$\bar{1},76261$	14	$\bar{1},69560$	25	$\bar{1},45471$	36	$\bar{2},83982$	47	$\bar{3},59365$
4	$\bar{1},77234$	15	$\bar{1},68038$	26	$\bar{1},41430$	37	$\bar{2},75105$	48	$\bar{3},49089$
5	$\bar{1},77760$	16	$\bar{1},66486$	27	$\bar{1},37032$	38	$\bar{2},65000$	49	$\bar{3},39225$
6	$\bar{1},77754$	17	$\bar{1},64892$	28	$\bar{1},32434$	39	$\bar{2},54705$	50	$\bar{3},31109$
7	$\bar{1},77458$	18	$\bar{1},63222$	29	$\bar{1},27810$	40	$\bar{2},44685$	51	$\bar{3},20781$
8	$\bar{1},76925$	19	$\bar{1},61459$	30	$\bar{1},22766$	41	$\bar{2},32144$	52	$\bar{3},06793$
9	$\bar{1},76147$	20	$\bar{1},59577$	31	$\bar{1},17570$	42	$\bar{2},18133$	53	$\bar{4},85274$
10	$\bar{1},75123$	21	$\bar{1},57582$	32	$\bar{1},11777$	43	$\bar{2},04495$		

TABLE VII.—*continued.*

Logarithm of the Carlisle chance of living 55 years for every age *a*.

<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.	<i>a</i>	$\lambda$ chance.
0	$\bar{1},60991$	10	$\bar{1},66949$	20	$\bar{1},43940$	30	$\bar{2},89693$	40	$\bar{3},77168$
1	$\bar{1},67464$	11	$\bar{1},65322$	21	$\bar{1},39887$	31	$\bar{2},81764$	41	$\bar{3},66198$
2	$\bar{1},70281$	12	$\bar{1},63646$	22	$\bar{1},35471$	32	$\bar{2},72872$	42	$\bar{3},56155$
3	$\bar{1},72278$	13	$\bar{1},61892$	23	$\bar{1},30840$	33	$\bar{2},62734$	43	$\bar{3},45869$
4	$\bar{1},72894$	14	$\bar{1},60051$	24	$\bar{1},26143$	34	$\bar{2},52392$	44	$\bar{3},36033$
5	$\bar{1},72914$	15	$\bar{1},58105$	25	$\bar{1},20979$	35	$\bar{2},42296$	45	$\bar{3},27966$
6	$\bar{1},72215$	16	$\bar{1},56072$	26	$\bar{1},15661$	36	$\bar{2},29634$	46	$\bar{3},17699$
7	$\bar{1},71169$	17	$\bar{1},53730$	27	$\bar{1},09743$	37	$\bar{2},15482$	47	$\bar{3},03735$
8	$\bar{1},69897$	18	$\bar{1},50967$	28	$\bar{1},03497$	38	$\bar{2},01689$	48	$\bar{4},82189$
9	$\bar{1},68490$	19	$\bar{1},47738$	29	$\bar{2},96773$	39	$\bar{3},89144$		

Logarithm of the Carlisle chance of living 60 years for every age *a*.

0	$\bar{1},56146$	9	$\bar{1},58982$	18	$\bar{1},29315$	27	$\bar{2},70839$	36	$\bar{3},63689$
1	$\bar{1},61925$	10	$\bar{1},57016$	19	$\bar{1},24615$	28	$\bar{2},60597$	37	$\bar{3},53503$
2	$\bar{1},63992$	11	$\bar{1},54908$	20	$\bar{1},19448$	29	$\bar{2},50196$	38	$\bar{3},43063$
3	$\bar{1},65251$	12	$\bar{1},52484$	21	$\bar{1},14119$	30	$\bar{2},40086$	39	$\bar{3},33077$
4	$\bar{1},65237$	13	$\bar{1},49638$	22	$\bar{1},08183$	31	$\bar{2},27417$	40	$\bar{3},24881$
5	$\bar{1},64740$	14	$\bar{1},46331$	23	$\bar{1},01902$	32	$\bar{2},13249$	41	$\bar{3},14535$
6	$\bar{1},63698$	15	$\bar{1},42467$	24	$\bar{2},95106$	33	$\bar{3},99425$	42	$\bar{3},00524$
7	$\bar{1},62349$	16	$\bar{1},38377$	25	$\bar{2},87906$	34	$\bar{3},86830$	43	$\bar{4},78568$
8	$\bar{1},60761$	17	$\bar{1},33950$	26	$\bar{2},79855$	35	$\bar{3},74779$		

Logarithm of the Carlisle chance of living 65 years for every age *a*.

0	$\bar{1},47972$	8	$\bar{1},48507$	16	$\bar{1},12608$	24	$\bar{2},48528$	32	$\bar{3},51270$
1	$\bar{1},53408$	9	$\bar{1},45261$	17	$\bar{1},06562$	25	$\bar{2},38298$	33	$\bar{3},40798$
2	$\bar{1},55171$	10	$\bar{1},41378$	18	$\bar{1},00378$	26	$\bar{2},25507$	34	$\bar{3},30763$
3	$\bar{1},56115$	11	$\bar{1},37213$	19	$\bar{2},93578$	27	$\bar{2},11216$	35	$\bar{3},22492$
4	$\bar{1},55729$	12	$\bar{1},32704$	20	$\bar{2},86374$	28	$\bar{3},97288$	36	$\bar{3},12025$
5	$\bar{1},54807$	13	$\bar{1},27986$	21	$\bar{2},78313$	29	$\bar{3},84634$	37	$\bar{4},97873$
6	$\bar{1},53285$	14	$\bar{1},23208$	22	$\bar{2},69278$	30	$\bar{3},72569$	38	$\bar{4},76162$
7	$\bar{1},51187$	15	$\bar{1},17975$	23	$\bar{2},59002$	31	$\bar{3},61471$		

Logarithm of the Carlisle chance of living 70 years for every age *a*.

0	$\bar{1},38039$	7	$\bar{1},31407$	14	$\bar{2},92171$	21	$\bar{2},23965$	28	$\bar{3},38661$
1	$\bar{1},42994$	8	$\bar{1},26855$	15	$\bar{2},84902$	22	$\bar{2},09655$	29	$\bar{3},28567$
2	$\bar{1},44010$	9	$\bar{1},22138$	16	$\bar{2},76802$	23	$\bar{3},95693$	30	$\bar{3},20281$
3	$\bar{1},43861$	10	$\bar{1},16886$	17	$\bar{2},67757$	24	$\bar{3},82967$	31	$\bar{3},09808$
4	$\bar{1},42008$	11	$\bar{1},11445$	18	$\bar{2},57478$	25	$\bar{3},70782$	32	$\bar{4},95640$
5	$\bar{1},39170$	12	$\bar{1},05416$	19	$\bar{2},47001$	26	$\bar{3},59561$	33	$\bar{4},73897$
6	$\bar{1},35590$	13	$\bar{2},99049$	20	$\bar{2},36767$	27	$\bar{3},49237$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 75 years for every age  $a$ .

$a$	$\lambda$ chance.	$a$	$\lambda$ chance.	$a$	$\lambda$ chance.	$a$	$\lambda$ chance.	$a$	$\lambda$ chance.
0	$\bar{1},22402$	6	$\bar{1},09821$	12	$\bar{2},66511$	18	$\bar{3},94169$	24	$\bar{3},26900$
1	$\bar{1},25299$	7	$\bar{1},04119$	13	$\bar{2},56149$	19	$\bar{3},81439$	25	$\bar{3},18494$
2	$\bar{1},24230$	8	$\bar{2},97918$	14	$\bar{2},45593$	20	$\bar{3},69250$	26	$\bar{3},07898$
3	$\bar{1},22209$	9	$\bar{2},91101$	15	$\bar{2},35295$	21	$\bar{3},58019$	27	$\bar{2},93607$
4	$\bar{1},18885$	10	$\bar{2},83813$	16	$\bar{2},22455$	22	$\bar{3},47676$	28	$\bar{2},71760$
5	$\bar{1},14678$	11	$\bar{2},75639$	17	$\bar{2},08134$	23	$\bar{3},37066$		

Logarithm of the Carlisle chance of living 80 years for every age  $a$ .

0	$\bar{2},97909$	5	$\bar{2},81604$	10	$\bar{2},34206$	15	$\bar{3},67778$	20	$\bar{3},16963$
1	$\bar{2},99530$	6	$\bar{2},74015$	11	$\bar{2},21291$	16	$\bar{3},56508$	21	$\bar{3},06356$
2	$\bar{2},96941$	7	$\bar{2},65214$	12	$\bar{2},06888$	17	$\bar{3},46155$	22	$\bar{2},92046$
3	$\bar{2},93272$	8	$\bar{2},55018$	13	$\bar{3},92839$	18	$\bar{3},35542$	23	$\bar{2},70166$
4	$\bar{2},87848$	9	$\bar{2},44523$	14	$\bar{3},80031$	19	$\bar{3},25372$		

Logarithm of the Carlisle chance of living 85 years for every age  $a$ .

0	$\bar{2},64836$	4	$\bar{2},41271$	8	$\bar{3},91708$	12	$\bar{3},44909$	16	$\bar{3},04845$
1	$\bar{2},63724$	5	$\bar{2},31997$	9	$\bar{3},78962$	13	$\bar{3},34213$	17	$\bar{2},90525$
2	$\bar{2},58037$	6	$\bar{2},19667$	10	$\bar{3},66689$	14	$\bar{3},23965$	18	$\bar{2},68641$
3	$\bar{2},50372$	7	$\bar{2},05591$	11	$\bar{3},55345$	15	$\bar{3},15490$		

Logarithm of the Carlisle chance of living 90 years for every age  $a$ .

0	$\bar{2},15229$	3	$\bar{3},87062$	6	$\bar{3},53721$	9	$\bar{3},22895$	12	$\bar{2},89279$
1	$\bar{2},09377$	4	$\bar{3},75709$	7	$\bar{3},43612$	10	$\bar{3},14401$	13	$\bar{2},67312$
2	$\bar{3},98414$	5	$\bar{3},64480$	8	$\bar{3},33082$	11	$\bar{3},03682$		

Logarithm of the Carlisle chance of living 95 years for every age  $a$ .

0	$\bar{3},47712$	2	$\bar{3},36435$	4	$\bar{3},19642$	6	$\bar{3},02058$	8	$\bar{2},66181$
1	$\bar{3},43431$	3	$\bar{3},28436$	5	$\bar{3},12193$	7	$\bar{2},87982$		

Logarithm of the Carlisle chance of living 100 years for every age  $a$ .

0	$\bar{4},95424$	1	$\bar{4},91768$	2	$\bar{4},80805$	3	$\bar{4},61535$
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TABLE VIII.—Logarithm of Deparcieux chance of living for every age *a*.

<i>a</i>	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0									
1									
2									
3	1̄,93450	1̄,89763	1̄,85126	1̄,80346	1̄,73957	1̄,62634	1̄,39967	2̄,85126	3̄,30103
4	1̄,94469	1̄,90644	1̄,85957	1̄,81188	1̄,74401	1̄,62495	1̄,37684	2̄,78408	3̄,01323
5	1̄,95159	1̄,91193	1̄,86455	1̄,81698	1̄,74418	1̄,61979	1̄,34747	2̄,70443	
6	1̄,95683	1̄,91575	1̄,86784	1̄,82040	1̄,74248	1̄,61130	1̄,31482	2̄,61130	
7	1̄,96027	1̄,91825	1̄,86981	1̄,82177	1̄,73928	1̄,59968	1̄,27663	2̄,50098	
8	1̄,96282	1̄,91985	1̄,87151	1̄,82222	1̄,73410	1̄,58512	1̄,23231	2̄,38721	
9	1̄,96495	1̄,92101	1̄,87278	1̄,82146	1̄,72821	1̄,56781	1̄,18415	2̄,25473	
10	1̄,96614	1̄,92122	1̄,87309	1̄,81970	1̄,72110	1̄,54688	1̄,12740	2̄,09691	
11	1̄,96582	1̄,92042	1̄,87239	1̄,81612	1̄,71269	1̄,52337	1̄,06380	3̄,90458	
12	1̄,96448	1̄,91860	1̄,87069	1̄,81067	1̄,70296	1̄,49545	2̄,99190	3̄,66454	
13	1̄,96313	1̄,91676	1̄,86896	1̄,80507	1̄,69184	1̄,46517	2̄,91676	3̄,36653	
14	1̄,96175	1̄,91488	1̄,86719	1̄,79932	1̄,68026	1̄,43215	2̄,83939	3̄,06854	
15	1̄,96034	1̄,91296	1̄,86539	1̄,79259	1̄,66820	1̄,39588	2̄,75284		
16	1̄,95892	1̄,91101	1̄,86357	1̄,78565	1̄,65447	1̄,35799	2̄,65447		
17	1̄,95798	1̄,90954	1̄,86150	1̄,77901	1̄,63941	1̄,31636	2̄,54071		
18	1̄,95703	1̄,90869	1̄,85940	1̄,77128	1̄,62230	1̄,26949	2̄,42439		
19	1̄,95606	1̄,90783	1̄,85651	1̄,76326	1̄,60286	1̄,21920	2̄,28978		
20	1̄,95508	1̄,90695	1̄,85356	1̄,75496	1̄,58074	1̄,16126	2̄,13077		
21	1̄,95460	1̄,90657	1̄,85030	1̄,74687	1̄,55755	1̄,09798	3̄,93876		
22	1̄,95412	1̄,90621	1̄,84619	1̄,73848	1̄,53097	1̄,02742	3̄,70006		
23	1̄,95363	1̄,90583	1̄,84194	1̄,72871	1̄,50204	2̄,95363	3̄,40340		
24	1̄,95313	1̄,90544	1̄,83757	1̄,71851	1̄,47040	2̄,87764	3̄,10679		
25	1̄,95262	1̄,90505	1̄,83225	1̄,70786	1̄,43554	2̄,79250			
26	1̄,95209	1̄,90465	1̄,82673	1̄,69555	1̄,39907	2̄,69555			
27	1̄,95156	1̄,90352	1̄,82103	1̄,68143	1̄,35838	2̄,58273			
28	1̄,95166	1̄,90237	1̄,81425	1̄,66527	1̄,31246	2̄,46736			
29	1̄,95177	1̄,90045	1̄,80720	1̄,64180	1̄,26314	2̄,33372			
30	1̄,95187	1̄,89848	1̄,79928	1̄,62566	1̄,20618	2̄,17569			
31	1̄,95197	1̄,89570	1̄,79227	1̄,60295	1̄,14338	3̄,98416			
32	1̄,95209	1̄,89207	1̄,78436	1̄,57685	1̄,07330	3̄,74594			
33	1̄,95220	1̄,88831	1̄,77508	1̄,54841	1̄,00000	3̄,44977			
34	1̄,95231	1̄,88444	1̄,76538	1̄,51727	2̄,92451	3̄,15366			
35	1̄,95243	1̄,87963	1̄,75524	1̄,48292	2̄,83988				
36	1̄,95256	1̄,87464	1̄,74346	1̄,44698	2̄,74346				
37	1̄,95196	1̄,86947	1̄,72937	1̄,40682	2̄,63117				
38	1̄,95071	1̄,86259	1̄,71361	1̄,36080	2̄,51570				
39	1̄,94868	1̄,85543	1̄,69503	1̄,31137	2̄,38195				
40	1̄,94661	1̄,84801	1̄,67379	1̄,25431	2̄,22382				
41	1̄,94373	1̄,84030	1̄,65098	1̄,19141	2̄,03219				
42	1̄,93998	1̄,83227	1̄,62476	1̄,12121	3̄,79385				
43	1̄,93611	1̄,82288	1̄,59621	1̄,04780	3̄,49757				
44	1̄,93213	1̄,81307	1̄,56496	2̄,97220	3̄,20135				
45	1̄,92720	1̄,80281	1̄,53049	2̄,88745					
46	1̄,92208	1̄,79090	1̄,49442	2̄,79090					
47	1̄,91751	1̄,77791	1̄,45486	2̄,67921					
48	1̄,91188	1̄,76290	1̄,41009	2̄,56439					
49	1̄,90675	1̄,74635	1̄,36269	2̄,43327					
50	1̄,90140	1̄,72718	1̄,30770	2̄,27721					
51	1̄,89657	1̄,70725	1̄,24768	2̄,08846					
52	1̄,89229	1̄,68478	1̄,18123	3̄,85387					

[continued.]





TABLE IX.—Logarithm of the Northampton chance for living at every age a.

a	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0	1̄,68764	1̄,64396	1̄,57564	1̄,49417	1̄,38958	1̄,24287	1̄,02428	2̄,60484	3̄,59643
1	1̄,81295	1̄,76713	1̄,69746	1̄,61431	1̄,50640	1̄,35435	1̄,12443	2̄,67151	3̄,59446
2	1̄,88378	1̄,83536	1̄,76454	1̄,67952	1̄,56809	1̄,41046	1̄,16788	2̄,67677	3̄,51790
3	1̄,91089	1̄,85979	1̄,78780	1̄,70070	1̄,58568	1̄,42229	1̄,16522	2̄,62961	3̄,37283
4	1̄,92894	1̄,87511	1̄,80190	1̄,71263	1̄,59383	1̄,42421	1̄,15070	2̄,55993	3̄,14495
5	1̄,93843	1̄,88180	1̄,80733	1̄,71581	1̄,59300	1̄,41691	1̄,12431	2̄,47370	4,80625
6	1̄,94739	1̄,88788	1̄,81211	1̄,71823	1̄,59118	1̄,40806	1̄,09339	2̄,37854	
7	1̄,95322	1̄,89101	1̄,81390	1̄,71755	1̄,58601	1̄,39522	1̄,05661	2̄,27263	
8	1̄,95660	1̄,89203	1̄,81352	1̄,71459	1̄,57827	1̄,37909	1̄,01505	2̄,15453	
9	1̄,95739	1̄,89080	1̄,81084	1̄,70923	1̄,56781	1̄,35940	2̄,99001	2,03386	
10	1̄,95632	1̄,88800	1̄,80653	1̄,70194	1̄,55523	1̄,33664	2̄,91720	3̄,90879	
11	1̄,95418	1̄,88451	1̄,80136	1̄,69345	1̄,54140	1̄,31148	2̄,85856	3̄,78151	
12	1̄,95158	1̄,88076	1̄,79574	1̄,68431	1̄,52668	1̄,28410	2̄,79299	3̄,63412	
13	1̄,94890	1̄,87691	1̄,78981	1̄,67479	1̄,51140	1̄,25433	2̄,71872	3,46194	
14	1̄,94617	1̄,87296	1̄,78369	1̄,66489	1̄,49527	1̄,22176	2̄,63099	3,21602	
15	1̄,94337	1̄,86890	1̄,77738	1̄,65457	1̄,47848	1̄,18588	2̄,53527	4,86782	
16	1̄,94049	1̄,86472	1̄,77084	1̄,64379	1̄,46067	1̄,14600	2,43115		
17	1̄,93779	1̄,86068	1̄,76433	1̄,63279	1̄,44200	1̄,10339	2,31941		
18	1̄,93543	1̄,85692	1̄,75799	1̄,62167	1̄,42249	1̄,05845	2,19793		
19	1̄,93341	1̄,85345	1̄,75184	1̄,61042	1̄,40201	1̄,01162	2,07647		
20	1̄,93168	1̄,85021	1̄,74562	1̄,59891	1̄,38032	2,96088	3,95247		
21	1̄,93033	1̄,84718	1̄,73927	1̄,58722	1̄,35730	2,90438	3,82733		
22	1̄,92918	1̄,84417	1̄,73274	1̄,57511	1̄,33253	2,84141	3,68255		
23	1̄,92801	1̄,84091	1̄,72589	1̄,56250	1̄,30543	2,76982	3,51304		
24	1̄,92679	1̄,83752	1̄,71872	1̄,54910	1̄,27559	2,68482	3,26984		
25	1̄,92553	1̄,83401	1̄,71120	1̄,53511	1̄,24251	2,59191	4,92445		
26	1̄,92423	1̄,83035	1̄,70330	1̄,52018	1̄,20551	2,49066			
27	1̄,92289	1̄,82654	1̄,69500	1̄,50421	1̄,16560	2,38162			
28	1̄,92149	1̄,82256	1̄,68624	1̄,48706	1̄,12302	2,26250			
29	1̄,92004	1̄,81843	1̄,67701	1̄,46860	1̄,07821	2,14306			
30	1̄,91853	1̄,81394	1̄,66723	1̄,44864	1̄,02920	2,02079			
31	1̄,91685	1̄,80894	1̄,65689	1̄,42697	2,97405	3,89700			
32	1̄,91498	1̄,80355	1̄,64592	1̄,40334	2,91223	3,75336			
33	1̄,91290	1̄,79788	1̄,63449	1̄,37742	2,84181	3,58503			
34	1̄,91073	1̄,79193	1̄,62231	1̄,34880	2,75802	3,34305			
35	1̄,90848	1̄,78567	1̄,60958	1̄,31698	2,66637	4,95892			
36	1̄,90612	1̄,77907	1̄,59595	1̄,28128	2,56643				
37	1̄,90365	1̄,77211	1̄,58132	1̄,24271	2,45873				
38	1̄,90107	1̄,76475	1̄,56557	1̄,20153	2,34101				
39	1̄,89839	1̄,75697	1̄,54856	1̄,15817	2,22302				
40	1̄,89541	1̄,74870	1̄,53011	1̄,11067	2,10226				
41	1̄,89209	1̄,74004	1̄,51012	1̄,05720	3,98015				
42	1̄,88857	1̄,73094	1̄,48836	2,99725	3,83838				
43	1̄,88498	1̄,72159	1̄,46452	2,92891	3,67213				
44	1̄,88120	1̄,71158	1̄,43807	2,84730	3,43232				
45	1̄,87719	1̄,70110	1̄,40850	2,75759	3,09044				
46	1̄,87295	1̄,68983	1̄,37516	2,66031					
47	1̄,86846	1̄,67767	1̄,33906	2,55508					
48	1̄,86368	1̄,66450	1̄,30046	2,43994					
49	1̄,85858	1̄,65017	1̄,25978	2,32463					
50	1̄,85329	1̄,63470	1̄,21526	2,20685					
51	1̄,84795	1̄,61803	1̄,16511	2,08806					
52	1̄,84237	1̄,59979	1,10868	3,94981					

[continued.]



How the value of particular assurances may be determined from the value of annuities, is shown in my Paper in the Philosophical Transactions for the year 1820, many of the cases of which are solved by methods essentially the same as those which have been long adopted; but when such assurances are but for terms, which are not of great extension, very near approximations may be had by using a geometrical progression, without confining the arithmetical operations to the same route, since the chance of extinction of the joint lives of the present age  $a, b, c, \&c.$  taking place between the period commencing with the time  $n+t-1$ , and finishing with the time  $n+t$ , from the present, is  $= (\overset{r}{\underset{1}{m}} \underline{a, b, c, \&c.} - \overset{r}{\underset{1}{n+t}} \underline{a, b, c, \&c.}) \div \overset{r}{\underset{1}{a, b, c, \&c.}}$ ; it follows that if  $r$  be the present value of unity, to be received certain in the time 1, and  $\overset{r}{\underset{1}{n+t-1}} \underline{a, b, c, \&c.} = \overset{r}{\underset{1}{n-1}} \underline{a, b, c, \&c.} \times \pi^t$ , whatever  $t$

may be, that  $\overset{r}{\underset{1}{m}} \underline{a, b, c, \&c.}$  or the assurance of unity to be received at the first of the equal periods 1, from the commencement of the time  $n-1$  to the expiration of the time  $m$ , which shall happen after the extinction of the joint lives, is equal to  $\frac{\overset{r}{\underset{1}{n-1}} \underline{a, b, c, \&c.}}{\overset{r}{\underset{1}{a, b, c, \&c.}}} \times \{ r^n \times (1-\pi) + r^{n+1} \times (\pi - \pi^2) + r^{n+2} \times (\pi^2 - \pi^3) \dots r^m \times (\pi^{m-n-1} - \pi^{m-n}) \} = (1-\pi) \times \frac{\overset{r}{\underset{1}{n-1}} \underline{a, b, c, \&c.}}{\overset{r}{\underset{1}{a, b, c, \&c.}}} \times \{ r^n + \pi r^{n+1} + \pi^2 r^{n+2} + \pi^3 r^{n+3} \dots r^m \pi^{m-n-1} \} = \frac{(1-\pi) r}{\overset{r}{\underset{1}{a, b, c, \&c.}}} \times \{ r^{n-1} \overset{r}{\underset{1}{n-1}} \underline{a, b, c, \&c.} + r^n \overset{r}{\underset{1}{n}} \underline{a, b, c, \&c.} + \dots + r^{m-1} \overset{r}{\underset{1}{m-1}} \underline{a, b, c, \&c.} \} = (1-\pi) \cdot r \times \overset{r}{\underset{1}{m-1}} \underline{a, b, c, \&c.}$

If the assurance be not deferred,  $n$  will be equal to 1, and we shall have, according to the hypothesis,  $\overset{r}{\underset{1}{m}} \underline{a, b, c, \&c.} = (1-\pi) \cdot r \times \overset{r}{\underset{0}{m}} \underline{a, b, c, \&c.}$ ; and also  $= \frac{1-\pi}{\pi} \cdot \overset{r}{\underset{1}{m}} \underline{a, b, c, \&c.}$ . If  $t$  be

taken equal to 1, we shall have from the equation  $L_{n+t-1:a, b, c, \&c.} =$   
 $L_{n-1:a, b, c, \&c.} \times \pi^t$ ,  $\pi = \frac{L_{n:a, b, c, \&c.}}{L_{n-1:a, b, c, \&c.}}$ , and this would be the real  
value which should be taken for  $\pi$ , if the geometrical pro-  
gression coincided perfectly with the fact; and it would be  
indifferent whether we made it equal to  $\frac{L_{n+t:a, b, c, \&c.}}{L_{n-1+t:a, b, c, \&c.}}$ , or  
 $\frac{L_{n:a, b, c, \&c.}}{L_{n-1:a, b, c, \&c.}}$ , as the two would be the same; but this not  
being the case, there will be a preference; and generally, if  
not always,  $\pi$  should be taken an intermediate value between  
the two; and when the term is not very long, it will answer  
a good purpose to take it about the middle between them,  
inclining generally, though perhaps not always, rather  
nearer the last than the first, as the first terms are generally  
of more consequence than the last. If the said assurance be  
not deferred, and instead of being paid for immediately, be  
to be paid for by equal periodic payments, at an unite of  
time from each other, up to the time  $m-1$  inclusive, and the  
first payment be to be made immediately, then will the  
present value of such periodic payment be  $\overset{r}{\underset{m-1}{\int}} a, b, c, \&c.$ , and  
consequently each payment, from what is shown above, is  
equal to  $\frac{\overset{r}{\int}_m a, b, c, \&c.}{\overset{r}{\int}_{m-1} a, b, c, \&c.} = (1-\pi) \cdot r$ . From whence  
we may draw an inference worthy of remark, namely; when  
an assurance of joint lives is meant to commence immedi-  
ately, and to continue for a term of  $t$  years, which is not  
large, and to be paid for by  $t$  annual payments, that those  
payments will not differ much with the increase of the time  
 $t$ , provided, as I have said, that  $t$  be not large, and the ages

be not at the extremes of life, a consequence which follows from the near agreement to a geometrical progression which takes place in the number of living at each small equal increment of time; that is to say, from the near coincidence of  $\frac{L}{n : a, b, c, \&c.}$  with  $\frac{L}{n+t : a, b, c, \&c.}$ , or the small variation of  $\pi$  for the different values of  $t$ : and also, that when the number of years for which an assurance continues be not very long, and the ages be not at the extremes of life, the annual premiums will not differ widely from the premiums to be paid for an assurance of one year of a life older than the proposed life by about half the term: thus, according to the Northampton table, at three per cent. to assure 100 *l.* at the

Age . . . . .	15	20	30	40	50	60	64
For 7 years, the annual premium by the common modes of calculation	£1..2..11	1.. 9.. 5	1..14..11	2.. 4.. 1	3.. 0.. 8	4.. 7.. 1	5.. 4..10
And the premium for one year assurance for an age 3 years older	1..3.. 3	1.. 9.. 8	1..15.. 0	2.. 4.. 6	3.. 1.. 0	4.. 7.. 8	5 5.. 6

the difference of which is very small.—As another example, let

Age . . . . .	10	20	30	40	50	60
For 10 years, the annual premium will be, by common modes of calculation	£0..19.. 2	1..9.. 1	1..15.. 8	2.. 5.. 8	3.. 3.. 4	4..12.. 6
Premium for one year assurance, age 5 years older	0..17..11	1..10.. 7	1..16.. 4	2.. 6.. 8	3.. 5.. 1	4..15.. 2

Here, except at the age 10, the excess is rather more in the approximation than in the first set of examples; but it should be recollected, that we took the exact middle, instead of inclining to the early age.

According to the Carlisle table of mortality at 3 per cent. to assure 100*l.* at the

Age . . . . .	10	20	30	40	50	60
For 7 years, the annual premium, by common modes of calculation . . . . .	£0 10 5	0 13 10	0 19 10	1 7 8	1 11 0	3 13 8
For one year, the premium . . . . .	0 10 5	0 13 9	0 19 2	1 8 6	1 12 1	3 15 9
For 10 years, the annual premium, by common modes of calculation . . . . .	0 11 3	0 14 7	1 0 4	1 7 7	1 14 11	3 17 8
For one year, at an age 5 years older . . . . .	0 12 0	0 14 2	0 19 11	1 9 0	1 14 10	3 19 9

Moreover, because  $\overset{r}{\underset{m-1}{\overline{1}}}_{a, b, c, \&c.}$ , or the single premium for the assurance of unity, on the joint lives *a, b, c, &c.* for *m* years, is  $= \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} \cdot r - \frac{\overset{r}{\underset{1}{\overline{1}}}_{a, b, c, \&c.}}{m} = \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} \cdot r + 1 -$

$$\frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} r^m - \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} = 1 - \frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} \cdot r^m - (1-r)^{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}};$$

if this be divided by  $\frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1}$ , we shall have the annual

premium for such assurance; that is,  $\frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} \cdot \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} =$

$$\frac{1 - \frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} \cdot r^m}{\frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1}} - 1 + r.$$

The said annual premium may be expressed by

$$\left(1 - \frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} \cdot r^m\right) \div \left(\frac{\overset{r}{\underset{1}{\overline{1}}}_{a-1, b-1, c-1, \&c.}}{m} \times \frac{L_{a-1, b-1, c-1, \&c.}}{r L_{a, b, c, \&c.}}\right) - 1 + r.$$

This last mode is well adapted to logarithms in the use of our general tables; and this method, supposing the annuities were accurately determinable by our general tables, would be accurate. The last formula is derived from that immediately before, in consequence of  $\frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1}$  being identical

with  $\frac{\overset{r}{\underset{1}{\overline{1}}}_{a-1, b-1, c-1, \&c.}}{m} \times \frac{L_{a-1, b-1, c-1, \&c.}}{r L_{a, b, c, \&c.}}$

Example. To find the annual premium to assure a life, at the age  $a$  years, for 10 years, according to the Carlisle mortality, and three per cent. interest.

$a =$	20	30	40	50	60	70
Log. of the accommoda. chance for living 10 yrs. at the age $a-1$ , Tab. V.	$\bar{1}.9690$	$\bar{1}.9556$	$\bar{1}.9406$	$\bar{1}.9328$	$\bar{1}.8435$	$\bar{1}.7005$
$\lambda 1,03^{-1} =$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$
Sum . . . =	$\bar{1}.8406$	$\bar{1}.8272$	$\bar{1}.8122$	$\bar{1}.8044$	$\bar{1}.7151$	$\bar{1}.5721$
Corresponding . . .	.91443	.90407	.89892	.89379	.84846	.78092
To this we get from Ta. I.	31	362	103	205	248	94
$\lambda \frac{1}{10} a-1$ . . . =	.91474	.90779	.90005	.89605	.85099	.78191
Therefore, $\lambda \frac{L+a+10}{L}$ (T. VII.)	$\bar{1}.96682$	$\bar{1}.95400$	$\bar{1}.93772$	$\bar{1}.91830$	$\bar{1}.81893$	$\bar{1}.59873$
$\lambda 1,03^{-10} =$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$
Sum = the log. . . .	$\bar{1}.83845$	$\bar{1}.82563$	$\bar{1}.80935$	$\bar{1}.78993$	$\bar{1}.69056$	$\bar{1}.47036$
The N° corresponding =	.68937	.66932	.64469	.61650	.49041	.29536
Its complement to unity	.31063	.33068	.35531	.38350	.50959	.70464
The log. of the last . =	$\bar{1}.49224$	$\bar{1}.51941$	$\bar{1}.55061$	$\bar{1}.58377$	$\bar{1}.70722$	$\bar{1}.84797$
Complement of $\lambda \frac{1}{10} a-1$ =	$\bar{1}.08526$	$\bar{1}.09221$	$\bar{1}.09995$	$\bar{1}.10395$	$\bar{1}.14901$	$\bar{1}.21809$
$\lambda \frac{L}{L-a+1}$ . . . . . =	$\bar{1}.99694$	$\bar{1}.99571$	$\bar{1}.99481$	$\bar{1}.99402$	$\bar{1}.98754$	$\bar{1}.97813$
$\lambda r^{-1}$ . . . . . =	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$
Sum = logarithm . . .	$\bar{2}.56160$	$\bar{2}.59449$	$\bar{2}.63253$	$\bar{2}.66890$	$\bar{2}.83093$	$\bar{1}.03135$
Number corresponding =	.03644	.03931	.04291	.04666	.06775	.10749
$1,03^{-1} 1$ . . . . . =	-.02913	-.02913	-.02913	-.02913	-.02913	-.02913
Ann. premium for an assurance of 1 <i>l.</i> . . . =	.00732	.01018	.01378	.01753	.03862	.07836
Ditto for 100 <i>l.</i> . . . . . =	£0..14..8	1.. 0.. 4	1.. 7.. 7	1..15.. 1	3..1 .. 3	7..16.. 9

} for annual premiums.

The reader has here an opportunity of comparing the results from my tables, with those above calculated by Mr. MILNE'S Carlisle tables.— I may probably be able at a future period to add examples, which I regret time will not at present permit.